Like-sign dimuon asymmetry of B^0 meson and LFV in $SU(5)$ SUSY GUT with S_4 flavor symmetry

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Abstract

The like-sign dimuon charge asymmetry of the B meson, which was reported in the DØ Collaboration, is studied in the $SU(5)$ SUSY GUT model with S_4 flavor symmetry. Additional CP violating effects from the squark sector are discussed in $B_s - \bar{B}_s$ mixing process. The predicted like-sign charge asymmetry is in the 2σ range of the combined result of DØ and CDF measurements. Since the SUSY contributions in the quark sector affect to the lepton sector because of the $SU(5)$ GUT relation, two predictions are given in the leptonic processes: (i) both BR($\mu \to e\gamma$) and the electron EDM are close to the present upper bound, (ii) the decay ratios of τ decays, $\tau \to \mu\gamma$ and $\tau \to e\gamma$, are related to each other via the Cabibbo angle λ_c: $\mathrm{BR}(\tau \to e\gamma)/\mathrm{BR}(\tau \to \mu\gamma) \simeq \lambda_c^2$. These are testable at future experiments.

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I. INTRODUCTION

The CP violation in the K and B_d mesons has been well explained within the framework of the standard model (SM). There is one phase, which is a unique source of the CP violation, so called Kobayashi-Maskawa (KM) phase [1], in the quark sector with three families. Until now, the KM phase has successfully described all data related with the CP violation of K and B_d systems.

However, there could be new sources of the CP violation if the SM is extended to the supersymmetric (SUSY) models. The CP violating phases appear in soft scalar mass matrices. These contribute to flavor changing neutral currents (FCNC) with the CP violation. Therefore, we should examine carefully CP violating phenomena in the quark sector.

The Tevatron experiments have searched possible effect of the CP violation in the B meson system [2, 3]. Recently, the DØ Collaboration reported the interesting result of the like-sign dimuon charge asymmetry $A^b_{sl}(\text{DØ}) = -(9.57 \pm 2.51 \pm 1.46) \times 10^{-3}$ [3]. This result is larger than the SM prediction $A^b_{sl}(\text{SM}) = (-2.3^{+0.5}_{-0.6}) \times 10^{-4}$ [4] at the 3.2σ level, which indicates an anomalous CP violating phase arising in the B_s meson mixing.

Actually, new physics have been discussed to explain the anomalous CP violation in several approaches. As a possibility, new physics contribute to decay width of the B_s meson [5]-[12]. Another possibility is to assume new physics does not give additional contribution to the decay width but the $B_s - \overline{B}_s$ mixing [13]-[26]. This typical model is the general SUSY model with gluino-mediated flavor and CP violation [14–18, 21, 22]. Relevant mass insertion (MI) parameters and/or squark mass spectrum can explain the anomalous CP violation in the B_s system. Since the squark flavor mixing is restricted in K and B_d meson systems, the systematic analyses are necessary to clarify the possible effect of squarks.

In this paper, we study the flavor and CP violation within the framework of the non-Abelian discrete symmetry [27] of quark and lepton flavors with SUSY. Then, the flavor symmetry controls the squark and slepton mass matrices as well as the quark and lepton ones. For example, the predicted squark mass matrices reflect structures of the quark mass matrices. Therefore, squark mass matrices provide us an important test for the flavor symmetry.

The non-Abelian discrete symmetry of flavors has been studied intensively in the quark and lepton sectors. Actually, the recent neutrino data analyses [28]-[31] indicate the tri-bimaximal mixing [32] -[35], which has been at first understood based on the non-Abelian finite group A_4 [36–40]. Until now, much progress has been made in the theoretical and phenomenological analysis of A_4 flavor model [41]- [111].

An attractive candidate of the flavor symmetry is the S_4 group, which was successful to explain both quark and lepton mixing [112]-[149]. Especially, S_4 flavor models to unify quarks and leptons have been proposed in the framework of the $SU(5)$ SUSY GUT [118–121], $SO(10)$ SUSY GUT [122–124], and the Pati-Salam SUSY GUT [125, 126]. These unified models seem to explain both mixing of quarks and leptons.

Some of us have studied S_4 flavor model [119], which gives the proper quark flavor mixing angles as well as the tri-bimaximal mixing of neutrino flavors. Especially, the Cabibbo angle is predicted to be 15° due to S_4 Clebsch-Gordan coefficients. Including the next-to-leading corrections of the S_4

symmetry, the predicted Cabibbo angle is completely consistent with the observed one.

We give the squark mass matrices in our S_4 flavor model by considering the gravity mediation within the framework of the supergravity theory. We estimate the SUSY breaking in the squark mass matrices by taking account of the next-to-leading S_4 invariant mass operators as well as the slepton mass matrices. Then, we can predict the CP violation in the B_s meson taking account of the constraints of the CP violation of K and B_d mesons. We also discuss the squark effect on $b \to s\gamma$ decay and the chromo–electric dipole moment (cEDM) .

Since our model is based on $SU(5)$ SUSY GUT, we can predict the lepton flavor violation (LFV), e.g., $\mu \to e\gamma$ and $\tau \to \mu\gamma$ processes [147]. In particular, the $\tau \to \mu\gamma$ decay ratio reflects the magnitude of the CP violation of the B_s meson.

This paper is organized as follows: In section 2, we discuss the possibility of new physics in the framework of the CP violation of the neutral B system. In section 3, we present briefly our S_4 flavor model of quarks and leptons in $SU(5)$ SUSY GUT, and present the squark and slepton mass matrices. In section 4, we discuss numerically the CP violation of the B_s meson with constraints of flavor and CP violations of K and B_d mesons. We also discuss the EDM of the electron, cEDM of strange quark and LFV. Section 5 is devoted to the summary. In appendices, we present relevant formulae in order to estimate the flavor violation and the CP violation.

II. $B_s - \bar{B}_s$ MIXING

In this section, we briefly discuss the theory and experimental results of the CP violation of the neutral B meson system. The effective Hamiltonian $\mathcal{H}^q_{\text{eff}} (q = d, s)$ of $B_q - \bar{B}_q$ system is given in terms of the dispersive (absorptive) part $M^q (\Gamma^q)$ as

$$\mathcal{H}^q_{\text{eff}} = M^q - \frac{i}{2}\Gamma^q, \tag{1}$$

where the off-diagonal elements M^q_{12} and Γ^q_{12} are responsible for the $B_q - \bar{B}_q$ oscillations. The light (L) and heavy (H) physical eigenstates $B^q_{L(H)}$ with mass $M^q_{L(H)}$ and the decay width $\Gamma^q_{L(H)}$ are obtained by diagonalizing the effective Hamiltonian $\mathcal{H}^q_{\text{eff}}$. The mass and decay width difference between B^q_L and B^q_H are related to the elements of $\mathcal{H}^q_{\text{eff}}$ as

$$\Delta M_q \equiv M^q_H - M^q_L = 2|M^q_{12}|, \qquad \Delta\Gamma_q \equiv \Gamma^q_L - \Gamma^q_H = 2|\Gamma^q_{12}| \cos\phi_q, \qquad \phi_q = \arg(-M^q_{12}/\Gamma^q_{12}), \tag{2}$$

where we have used $\Delta\Gamma_q \ll \Delta M_q$.

The "wrong-sign" charge asymmetry a^q_{sl} of $B_q \to \mu^- X$ decay is defined as

$$a^q_{sl} \equiv \frac{\Gamma(\bar{B}_q \to \mu^+ X) - \Gamma(B_q \to \mu^- X)}{\Gamma(\bar{B}_q \to \mu^+ X) + \Gamma(B_q \to \mu^- X)} \simeq \text{Im}\left(\frac{\Gamma^q_{12}}{M^q_{12}}\right) = \frac{|\Gamma^q_{12}|}{|M^q_{12}|} \sin\phi_q \ . \tag{3}$$

The like-sign dimuon charge asymmetry A^b_{sl} is defined and related with a^q_{sl} as [150]

$$A^b_{sl} \equiv \frac{N^{++}_b - N^{--}_b}{N^{++}_b + N^{--}_b} = (0.506 \pm 0.043) a^d_{sl} + (0.494 \pm 0.043) a^s_{sl}, \tag{4}$$

where $N_b^{\pm\pm}$ is the number of events of $b\bar{b} \to \mu^\pm\mu^\pm X$.

The SM prediction of A_{sl}^b is given as [4]

$$A_{sl}^b(\mathrm{SM}) = (-2.3^{+0.5}_{-0.6}) \times 10^{-4}, \tag{5}$$

which is calculated from [4] [1]

$$a_{sl}^d(\mathrm{SM}) = (-4.8^{+1.0}_{-1.2}) \times 10^{-4}, \quad a_{sl}^s(\mathrm{SM}) = (2.06 \pm 0.57) \times 10^{-5}. \tag{6}$$

Recently, the DØ collaboration reported A_{sl}^b with 6.1 fb^{-1} data set as [3]

$$A_{sl}^b(\mathrm{D\O}) = -(9.57 \pm 2.51 \pm 1.46) \times 10^{-3}, \tag{7}$$

which shows 3.2 σ deviation from the SM prediction of Eq.(5). On the other hand, the result by the CDF collaboration with 1.6 fb^{-1} data [2] $A_{sl}^b(\mathrm{CDF}) = (8.0 \pm 9.0 \pm 6.8) \times 10^{-3}$ is consistent with the SM prediction while it has large errors. Combining these measurements, one can obtain

$$A_{sl}^b(\mathrm{CDF} + \mathrm{D\O}) = -(8.5 \pm 2.8) \times 10^{-3}, \tag{8}$$

which is still 3 σ away from the SM prediction.

The DØ Collaboration have performed the direct measurement of a_{sl}^s [152] as $a_{sl}^s(\mathrm{D\O}) = -(1.7 \pm 9.1^{+1.4}_{-1.5}) \times 10^{-3}$, which is consistent with the SM prediction because of its large errors. However, if one use the present experimental value of a_{sl}^d [3, 153, 154], $a_{sl}^d(\exp) = -(4.7 \pm 4.6) \times 10^{-3}$, one can find that [3, 154]

$$a_{sl}^s = -0.0146 \pm 0.0075, \tag{9}$$

is required to obtain $A_{sl}^b(\mathrm{D\O})$. The central value of the required $|a_{sl}^s|$ is about three orders of magnitude larger than the SM prediction $a_{sl}^s(\mathrm{SM})$. Combining all results, one can obtain the average value

$$a_{sl}^s(\mathrm{average}) \simeq -(12.7 \pm 5.0) \times 10^{-3}, \tag{10}$$

which is still 2.5 σ away from the SM prediction $a_{sl}^s(\mathrm{SM})$. Therefore, if the DØ result is confirmed, it is a promising hint of new physics (NP) beyond the SM.

The contribution of NP to the dispersive part of the Hamiltonian is parameterized as

$$M_{12}^q = M_{12}^{q,SM} + M_{12}^{q,NP} = M_{12}^{q,SM}\left(1 + h_q e^{2i\sigma_q}\right) = M_{12}^{q,SM}\Delta_q, \quad \Delta_q = |\Delta_q|\, e^{i\phi_{\Delta_q}}, \tag{11}$$

where the SM contribution $M_{12}^{q,SM}$ is given by

$$M_{12}^{q,SM} = \frac{G_F^2 M_{B_q}}{12\pi^2} M_W^2 (V_{tb}V_{tq}^*)^2 \hat{\eta}_B S_0(x_t) f_{B_q}^2 B_q, \tag{12}$$

[1] Recently, the SM predictions are updated [151] by the same authors. However in this paper, we use the widely-accepted results of Ref. [4].

Input		Input	
f_{B_s}	$(231 \pm 3 \pm 15)$ MeV	$B_s(m_b)$	$0.841 \pm 0.013 \pm 0.020$
f_{B_s}/f_{B_d}	$1.209 \pm 0.007 \pm 0.023$	B_s/B_d	$1.01 \pm 0.01 \pm 0.03$
$\hat{\eta}_B$	0.8393 ± 0.0034	$S_0(x_t)$	2.35
M_s	5.3663 ± 0.0006 GeV	$\Delta M_s^{\rm exp}$	$17.77 \pm 0.10 \pm 0.07$ ps^{-1}
M_d	5.27917 ± 0.00029 GeV	$\Delta M_d^{\rm exp}$	0.507 ± 0.005 ps^{-1}
$m_d(m_b)$	$(5.1 \pm 1.3) \times 10^{-3}$ GeV	$m_s(m_b)$	0.085 ± 0.017 GeV
$m_b(m_b)$	4.248 ± 0.051 GeV	$\Delta\Gamma_s^{SM}$	(0.096 ± 0.039) ps^{-1}
$\phi_{d,SM}$	$\left(-10.1^{+3.7}_{-6.3}\right) \times 10^{-2}$	$\phi_{s,SM}$	$\left(+7.4^{+0.8}_{-3.2}\right) \times 10^{-3}$
$\lvert\Gamma_{12}^{s,SM}\rvert/\lvert M_{12}^{s,SM}\rvert$	$(4.97 \pm 0.94) \times 10^{-3}$	$\Delta\Gamma_d^{SM}/\Delta M_d^{SM}$	$\left(52.6^{+11.5}_{-12.8}\right) \times 10^{-4}$

TABLE I: Parameters of the neutral B meson mixing and quark masses [4, 155].

with parameters listed in Table 1.

Using these parameters, the mass difference of B_q meson, ΔM_q, is given by

$$\Delta M_q = \Delta M_q^{SM} \left|1 + h_q e^{2i\sigma_q}\right| = \Delta M_q^{SM} \left|\Delta_q\right|. \tag{13}$$

Since the SM contribution to the absorptive part Γ_{12}^s is dominated by tree-level decay $b \to c\bar{c}s$, one can set $\Gamma_{12}^s = \Gamma_{12}^{s,SM}$. In this case, the wrong-sign charge asymmetry a_{sl}^s is written as

$$a_{sl}^q = \frac{|\Gamma_{12}^{q,SM}|}{|M_{12}^{q,SM}|} \frac{\sin\left(\phi_{q,SM} + \phi_{\Delta q}\right)}{|\Delta_q|}. \tag{14}$$

Taking the experimental value $\Delta M_s^{\rm exp}$ into account, one finds that $|\Delta_s|$ is strongly constrained in the region $|\Delta_s| = 0.92 \pm 0.32$ [4]. Therefore, unphysical condition $\sin(\phi_{s,SM} + \phi_{\Delta s}) = -2.56 \pm 1.16$ is required to obtain 1 σ range of the charge asymmetry (See also [156]). Also as discussed in Ref [157], by using the SM prediction of $\Gamma_{12}^{d,s}$ and experimental values of $\Delta M_{d,s}$, they found in model-independent way that the like-sign charge asymmetry is bounded as $-A_{sl}^b < 3.16 \times 10^{-3}$, where the CP violation $S_{J/\psi K_S}$ and $S_{J/\psi\phi}$ are also taken into account.

Now we discuss how to avoid this unphysical condition to obtain large charge asymmetry. As the first possibility, one can consider the NP contributions to Γ_{12}^s, which come from additional contributions to decay processes $b \to c\bar{c}s$, $\tau^+\tau^- s$, etc. By using the DØ and CDF experimental data of $B_s \to J/\psi\phi$ decay [158], one can subtract $\Delta\Gamma_s$ and $\beta_s^{J/\psi\phi} \simeq -\phi_s/2$ as [153] [2]

$$\Delta\Gamma_s = \pm(0.154^{+0.054}_{-0.070})\ {\rm ps}^{-1}, \qquad \beta_s^{J/\psi\phi} = (0.39^{+0.18}_{-0.14}) \text{ or } (1.18^{+0.14}_{-0.18}), \tag{15}$$

where the sign of $\Delta\Gamma_s$ is still undetermined, and positive (negative) sign corresponds to the first (second) region of $\beta_s^{J/\psi\phi}$. Comparing them with the SM predictions, one finds that there still can

[2] See also Ref.[154] for recent results.

exist additional contributions to Γ_{12}^s from NP. This possibility has been studied in several models[3] [5–12].

In Ref.[13], while there are no NP contributions to Γ_{12}^s in their model, they employed the experimental value of $\Delta\Gamma_s$ of Eq.(15) since there must exist theoretical uncertainties. In the (h_q, σ_q) parametrization of NP, the best fit values of (h_s, σ_s) are obtained as [162]

$$(h_s, \sigma_s) \simeq (0.5, 120^\circ),\ (1.8, 100^\circ), \tag{16}$$

by taking ΔM_q, $\Delta\Gamma_q$, $S_{\psi K}$ and $S_{J/\psi\phi}$ into account, with varying $|\Gamma_{12}^s|$ in the range $0 - 0.3\text{ps}^{-1}$. In that paper, one can read that the region of $h_d \lesssim h_s$ is favored as seen in Refs. [14–16].

However in ordinary SUSY models, gluino-squark box diagrams do not give additional contributions to Γ_{12}^s since such diagrams do not generate additional decay modes of bottom quark. Therefore as the other possibility, constraint for ΔM_q is relaxed in Refs.[17, 18]. In those papers, they consider models that NP does not give additional contributions to Γ_{12}^s, but to M_{12}^q. They take a conservative constraint $0.6 < \Delta M_{d,s}/\Delta M_{d,s}^{\text{exp}} < 1.4$ [17] and the UTfit [19] allowed region $0.776 < \Delta M_{d,s}/\Delta M_{d,s}^{SM} < 1.162$ [18]. See also Refs. [20–22, 24, 25, 157] for other possibilities.

In this paper, we consider the NP contribution to $B_s - \bar{B}_s$ mixing by gluino-squark box diagrams in a $SU(5)$ SUSY GUT model with S_4 flavor symmetry. As shall be discussed in the next section, the soft SUSY breaking terms and related MI parameters $(\delta_d^{AB})_{ij} (A, B = L, R)$ obey S_4 flavor symmetry. In such SUSY models, there are no new contributions to Γ_{12}^s [14–18, 21, 22]. While the SUSY contributions to $B_s - \bar{B}_s$ mixing are induced by $(\delta_d^{AB})_{23}$, it is constrained by $b \to s\gamma$ decay. Since the other MI parameters of down-type squark sector are related to $(\delta_d^{AB})_{23}$ due to S_4 symmetry, K and B_d meson mixing, which are affected by $(\delta_d^{AB})_{12}$ and $(\delta_d^{AB})_{13}$, respectively, should also be taken into account. The CP violation in B_s meson system is related to cEDM of the strange quark d_s^C as well. Moreover, the leptonic processes such as $\tau \to \mu\gamma$ affected by $(\delta_\ell^{AB})_{23}$ should also be taken into account due to $SU(5)$ GUT relation.

Taking the above processes into account, we assume the following conditions in our numerical calculation: (i) the meson mass differences satisfy

$$0.6 < \frac{\Delta M_{d,s}}{\Delta M_{d,s}^{\text{exp}}} < 1.4, \qquad \frac{|M_{12}^{K,SUSY}|}{\Delta M_K^{\text{exp}}} < 1, \qquad \frac{|\text{Im} M_{12}^{K,SUSY}|}{\sqrt{2}\Delta M_K^{\text{exp}}} < \epsilon_K = 2.2 \times 10^{-3}, \tag{17}$$

(ii) cEDM of the strange quark is constrained by the neutron EDM as [163, 164]

$$|e d_s^C| < 1.0 \times 10^{-25}\ e\text{cm}, \tag{18}$$

(iii) the NP contribution to the branching ratio (BR) of $b \to s\gamma$ is constrained as

$$\text{BR}(b \to s\gamma)^{NP} \ <\ 1.0 \times 10^{-4}. \tag{19}$$

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[3] However, the NP contributions to Γ_{12}^q will be strongly constrained by the lifetime ratio τ_{B_s}/τ_{B_d}. We would like to thank A. Lenz for pointing out this point.

	(T_1,T_2)	T_3	(F_1,F_2,F_3)	(N_e^c,N_μ^c)	N_τ^c	H_5	$H_{\bar{5}}$	H_{45}	Θ
$SU(5)$	10	10	$\bar{5}$	1	1	5	$\bar{5}$	45	1
S_4	**2**	**1**	**3**	**2**	$\mathbf{1'}$	**1**	**1**	**1**	**1**
Z_4	$-i$	-1	i	1	1	1	1	-1	1
$U(1)_{FN}$	1	0	0	1	0	0	0	0	-1

	(χ_1,χ_2)	(χ_3,χ_4)	(χ_5,χ_6,χ_7)	$(\chi_8,\chi_9,\chi_{10})$	$(\chi_{11},\chi_{12},\chi_{13})$	χ_{14}
$SU(5)$	1	1	1	1	1	1
S_4	**2**	**2**	$\mathbf{3'}$	**3**	**3**	**1**
Z_4	$-i$	1	$-i$	-1	i	i
$U(1)_{FN}$	-1	-2	0	0	0	-1

TABLE II: Assignments of $SU(5)$, S_4, Z_4, and $U(1)_{FN}$ representations.

While the upper bounds of LFV decay processes $\ell_i \to \ell_j\gamma$ and the electron EDM are given by [165, 166]

$$\mathrm{BR}(\mu \to e\gamma) < 1.2\times 10^{-11}, \qquad \mathrm{BR}(\tau \to \mu\gamma) < 4.4\times 10^{-8}, \tag{20}$$

$$\mathrm{BR}(\tau \to e\gamma) < 3.3\times 10^{-8}, \qquad |ed_e| < 1.6\times 10^{-27}\ e\mathrm{cm}, \tag{21}$$

we do not take these bounds into account in the numerical calculation below. Instead, in the allowed parameter region of our model which can explain the like-sign charge asymmetry, we will obtain the predictions for LFV processes.

We perform numerical analysis in the section IV after introducing the S_4 flavor model in the next section.

III. THE S_4 FLAVOR MODEL

We briefly review S_4 flavor model of quarks and leptons, which was proposed in [119]. As the model is based on $SU(5)$ SUSY GUT, it gives sfermion mass matrices as well as quark and lepton mass matrices.

A. CKM Mixing

In the $SU(5)$ GUT, matter fields are unified into 10 and $\bar{5}$-dimensional representations as $10 \subset (Q, u^c, e^c)$ and $\bar{5} \subset (d^c, L)$. Three generations of $\bar{5}$, which are denoted by F_i $(i = 1, 2, 3)$, are assigned to **3** of S_4. On the other hand, the third generation of the 10-dimensional representation, T_3, is assigned to **1** of S_4, and the first and second generations of 10, (T_1, T_2), are assigned to **2** of S_4, respectively. Right-handed neutrinos, which are $SU(5)$ gauge singlets, are also assigned to **2** for

the first and second generations, (N_e^c, N_μ^c), and $\mathbf{1}'$ for the third one, N_τ^c. The 5-dimensional, $\bar{5}$-dimensional, and 45-dimensional Higgs of $SU(5)$, H_5, $H_{\bar{5}}$, and H_{45} are assigned to $\mathbf{1}$ of S_4. In order to obtain desired mass matrices, we introduce $SU(5)$ gauge singlets χ_i, so called flavons, which couple to quarks and leptons.

The Z_4 symmetry is added to obtain relevant couplings. Further, the Froggatt-Nielsen mechanism [167] is introduced to get the natural hierarchy among quark and lepton masses, as an additional $U(1)_{FN}$ flavor symmetry, where Θ denotes the Froggatt-Nielsen flavon. The particle assignments of $SU(5)$, S_4, Z_4, and $U(1)_{FN}$ are presented in Table II.

The couplings of flavons with fermions are restricted as follows. At the leading order, (χ_3, χ_4) are coupled with the right-handed Majorana neutrino sector, (χ_5, χ_6, χ_7) are coupled with the Dirac neutrino sector, $(\chi_8, \chi_9, \chi_{10})$ and $(\chi_{11}, \chi_{12}, \chi_{13})$ are coupled with the charged lepton and down-type quark sectors. At the next-to-leading order, (χ_1, χ_2) are coupled with the up-type quark sector, and χ_{14} contributes to the charged lepton and down-type quark sectors, and then the mass ratio of the electron and down quark is reproduced properly.

Our model predicts the quark mixing as well as the tri-bimaximal mixing of leptons. Especially, the Cabibbo angle is predicted to be 15° at the leading order. The model is consistent with the observed CKM mixing angles and CP violation as well as the non-vanishing U_{e3} of the neutrino flavor mixing.

Let us write down the superpotential respecting S_4, Z_4 and $U(1)_{FN}$ symmetries in terms of the S_4 cutoff scale Λ, and the $U(1)_{FN}$ cutoff scale $\overline{\Lambda}$. In our calculation, both cutoff scales are taken as the GUT scale which is around 10^{16}GeV. The $SU(5)$ invariant superpotential of the Yukawa sector up to the linear terms of χ_i $(i = 1, \cdots, 13)$ is given as

$$\begin{aligned} w = {} & y_1^u (T_1, T_2) \otimes T_3 \otimes (\chi_1, \chi_2) \otimes H_5/\Lambda + y_2^u T_3 \otimes T_3 \otimes H_5 \\ & + y_1^N (N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes \Theta^2/\bar{\Lambda} \\ & + y_2^N (N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes (\chi_3, \chi_4) + M N_\tau^c \otimes N_\tau^c \\ & + y_1^D (N_e^c, N_\mu^c) \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_5 \otimes \Theta/(\Lambda\bar{\Lambda}) \\ & + y_2^D N_\tau^c \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_5/\Lambda \\ & + y_1 (F_1, F_2, F_3) \otimes (T_1, T_2) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes H_{45} \otimes \Theta/(\Lambda\bar{\Lambda}) \\ & + y_2 (F_1, F_2, F_3) \otimes T_3 \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{\bar{5}}/\Lambda, \end{aligned} \tag{22}$$

where y_1^u, y_2^u, y_1^N, y_2^N, y_1^D, y_2^D, y_1, and y_2 are Yukawa couplings of order one, and M is the right-handed Majorana mass, which is taken to be 10^{12}GeV.

In order to predict the desired quark and lepton mass matrices, we require vacuum alignments for the vacuum expectation values (VEV's) of flavons. According to the potential analysis, which was presented in [119], we have conditions of VEV's to realize the potential minimum $(V = 0)$ as follows:

$$\begin{aligned} & (\chi_1, \chi_2) = (1, 1), \quad (\chi_3, \chi_4) = (0, 1), \quad (\chi_5, \chi_6, \chi_7) = (1, 1, 1), \quad (\chi_8, \chi_9, \chi_{10}) = (0, 1, 0), \\ & (\chi_{11}, \chi_{12}, \chi_{13}) = (0, 0, 1), \quad \chi_{14}^2 = -\frac{2\eta_2}{\eta_3}\chi_1^2, \end{aligned} \tag{23}$$

where these magnitudes are given in arbitrary units. Hereafter, we suppose these gauge-singlet scalars develop VEV's by denoting $\langle \chi_i \rangle = a_i \Lambda$.

Denoting Higgs doublets as h_u and h_d, we take VEV's of following scalars by

$$\langle h_u \rangle = v_u, \quad \langle h_d \rangle = v_d, \quad \langle h_{45} \rangle = v_{45}, \quad \langle \Theta \rangle = \theta, \tag{24}$$

which are supposed to be real. We define $\lambda \equiv \theta/\Lambda$ to describe the Froggatt-Nielsen mechanism [4].

Now, we can write down quark and lepton mass matrices by using the S_4 multiplication rule in Appendix A. The down-type quark mass matrix at the leading order is given as

$$M_d = \begin{pmatrix} 0 & 0 & 0 \\ y_1 \lambda a_9 v_{45}/\sqrt{2} & y_1 \lambda a_9 v_{45}/\sqrt{6} & 0 \\ 0 & 0 & y_2 a_{13} v_d \end{pmatrix} . \tag{25}$$

Then, we have

$$M_d^\dagger M_d = v_d^2 \begin{pmatrix} \frac{1}{2}|\bar{y}_1 \lambda a_9|^2 & \frac{1}{2\sqrt{3}}|\bar{y}_1 \lambda a_9|^2 & 0 \\ \frac{1}{2\sqrt{3}}|\bar{y}_1 \lambda a_9|^2 & \frac{1}{6}|\bar{y}_1 \lambda a_9|^2 & 0 \\ 0 & 0 & |y_2|^2 a_{13}^2 \end{pmatrix} , \tag{26}$$

where we denote $\bar{y}_1 v_d = y_1 v_{45}$. This matrix can be diagonalized by the orthogonal matrix $U_d^{(0)}$ as

$$U_d^{(0)} = \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{27}$$

The down-type quark masses are given as

$$m_d^2 = 0 \ , \quad m_s^2 = \frac{2}{3}|\bar{y}_1 \lambda a_9|^2 v_d^2 \ , \quad m_b^2 = |y_2|^2 a_{13}^2 v_d^2 \ . \tag{28}$$

The down quark mass vanishes, however tiny masses appear at the next-to-leading order.

The relevant superpotential of down sector at the next-to-leading order is given as

$$\begin{aligned} \Delta w_d = & \, y_{\Delta_a} (T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_1, \chi_2) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{\bar{5}}/\Lambda^2 \\ & + y_{\Delta_b} (T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes \chi_{14} \otimes H_{\bar{5}}/\Lambda^2 \\ & + y_{\Delta_c} (T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_1, \chi_2) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_{45}/\Lambda^2 \\ & + y_{\Delta_d} (T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_{14} \otimes H_{45}/\Lambda^2 \\ & + y_{\Delta_e} T_3 \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes H_{\bar{5}} \otimes /\Lambda^2 \\ & + y_{\Delta_f} T_3 \otimes (F_1, F_2, F_3) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{45} \otimes /\Lambda^2 \ , \end{aligned} \tag{29}$$

which gives the correction terms in the down-type quark mass matrix.

The down-type quark mass matrix including the next-to-leading order is

$$M_d \simeq \begin{pmatrix} \bar{\epsilon}_{11} & \bar{\epsilon}_{21} & \bar{\epsilon}_{31} \\ \frac{\sqrt{3}m_s}{2} + \bar{\epsilon}_{12} & \frac{m_s}{2} + \bar{\epsilon}_{22} & \bar{\epsilon}_{32} \\ \bar{\epsilon}_{13} & \bar{\epsilon}_{23} & m_b + \bar{\epsilon}_{33} \end{pmatrix} , \tag{30}$$

[4] Notice that this λ is not related to the Cabibbo angle λ_c in our model.

where the explicit forms of $\bar{\epsilon}_{ij}$'s are given in Appendix B, and m_s and m_b are given in Eq. (28). This mass matrix can be diagonalized by

$$M_d^{\rm diag} = V_d^\dagger M_d U_d^{(0)} U_d^{(1)}, \tag{31}$$

where mixing matrices for left-hand $U_d^{(1)}$ and for right-hand V_d are estimated as

$$\begin{aligned} U_d^{(1)} &= \begin{pmatrix} 1 & \theta_{12}^d & \theta_{13}^d \\ -\theta_{12}^d - \theta_{13}^d\theta_{23}^d & 1 & \theta_{23}^d \\ -\theta_{13}^d + \theta_{12}^d\theta_{23}^d & -\theta_{23}^d - \theta_{12}^d\theta_{13}^d & 1 \end{pmatrix}, \\ V_d &= \begin{pmatrix} 1 & \frac{\tilde{a}}{\lambda} & \tilde{a} \\ -\frac{\tilde{a}}{\lambda} - \tilde{a}^2 & 1 & \tilde{a} \\ -\tilde{a} + \frac{\tilde{a}^2}{\lambda} & -\tilde{a} - \frac{\tilde{a}^2}{\lambda} & 1 \end{pmatrix}, \end{aligned} \tag{32}$$

where $\tilde{a}$ denotes the typical value of the square root of the relevant sum of $a_i a_j$'s as discussed in the next subsection. We neglect CP violating phases then mixing angles $\theta_{12}^d,\ \theta_{13}^d,\ \theta_{23}^d$ are given as

$$\theta_{12}^d = \mathcal{O}\left(\frac{m_d}{m_s}\right) = \mathcal{O}\left(0.05\right), \quad \theta_{13}^d = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}\left(0.005\right), \quad \theta_{23}^d = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}\left(0.005\right). \tag{33}$$

On the other hand, the superpotential of up sector at the next-to-leading order is

$$\begin{aligned} \Delta w_u = {} & y_{\Delta_a}^u (T_1, T_2) \otimes (T_1, T_2) \otimes (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes H_5/\Lambda^2 \\ & + y_{\Delta_b}^u (T_1, T_2) \otimes (T_1, T_2) \otimes \chi_{14} \otimes \chi_{14} \otimes H_5/\Lambda^2 \\ & + y_{\Delta_c}^u T_3 \otimes T_3 \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes H_5/\Lambda^2. \end{aligned} \tag{34}$$

Then the mass matrix becomes

$$M_u = v_u \begin{pmatrix} 2y_{\Delta_{a_1}}^u a_1^2 + y_{\Delta_b}^u a_{14}^2 & y_{\Delta_{a_2}}^u a_1^2 & y_1^u a_1 \\ y_{\Delta_{a_2}}^u a_1^2 & 2y_{\Delta_{a_1}}^u a_1^2 + y_{\Delta_b}^u a_{14}^2 & y_1^u a_1 \\ y_1^u a_1 & y_1^u a_1 & y_2^u + y_{\Delta_c}^u a_9^2 \end{pmatrix}. \tag{35}$$

This symmetric mass matrix is diagonalized by the unitary matrix U_u as

$$U_u = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & r_c \\ 0 & -r_c & r_t \end{pmatrix}, \tag{36}$$

where $r_c = \sqrt{m_c/(m_c + m_t)}$ and $r_t = \sqrt{m_t/(m_c + m_t)}$.

Therefore, the CKM matrix V can be written as[5]

$$V = U_u^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\rho} & 0 \\ 0 & 0 & 1 \end{pmatrix} U_d^{(0)} U_d^{(1)}, \tag{37}$$

[5] The renormalization group effect for the CKM matrix is small so that the matrix given in the text can be regarded as the one at the electroweak scale.

where the phase ρ is an arbitrary parameter originating from complex Yukawa couplings.

At the leading order, the Cabibbo angle is derived as $60^\circ - 45^\circ = 15^\circ$ and it can be naturally fitted to the observed value by including the next-to-leading order as follows:

$$V_{us} \simeq \theta_{12}^d \cos 15^\circ + \sin 15^\circ. \tag{38}$$

The V_{cb} and V_{ub} mixing elements are expressed as

$$\begin{aligned} V_{ud} &\simeq \cos 15^\circ - (\theta_{12}^d + \theta_{13}^d\theta_{23}^d)\sin 15^\circ, \\ V_{cb} &\simeq -r_t\theta_{13}^d e^{i\rho}\sin 15^\circ + r_t\theta_{23}^d e^{i\rho}\cos 15^\circ - r_c\ , \\ V_{ub} &\simeq \theta_{13}^d\cos 15^\circ + \theta_{23}^d \sin 15^\circ, \end{aligned} \tag{39}$$

which are consistent with observed values.

We can also predict mass matrices of the charged leptons and neutrinos, which give the tri-bimaximal mixing of leptons. Details are shown in Appendix C.

Since mass eigenvalues of quarks and leptons are give in terms of $a_i = \langle\chi_i\rangle/\Lambda$, we can estimate a_i by putting the observed quark and lepton masses. These are given as

$$\begin{aligned} &a_3 = a_8 = a_{10} = a_{11} = a_{12} = 0, \qquad a_1 = a_2 \simeq \sqrt{\frac{m_c}{2\left|y^u_{\Delta_{a_2}} - \frac{y_1^{u2}}{y_2^u}\right| v_u}}\ , \\ &a_4 = \frac{(y_1^D\lambda)^2(m_{\nu_3}-m_{\nu_1})m_{\nu_2}M}{6y_2^N {y_2^D}^2 m_{\nu_1}m_{\nu_3}\Lambda}, \qquad a_5 = a_6 = a_7 = \frac{\sqrt{m_{\nu_2}M}}{\sqrt{3}y_2^D v_u}, \\ &a_9 = \frac{m_\mu}{\sqrt{6}|\bar{y}_1|\lambda v_d}, \qquad a_{13} = \frac{m_\tau}{y_2 v_d}\ , \end{aligned} \tag{40}$$

where masses of quarks and leptons are given at the GUT scale, and the light neutrino masses $m_{\nu_{1,2,3}}$ are given in the Appendix C. Hereafter, we take $\lambda = 0.1$ in our calculations.

B. Squark and slepton mass matrices

Here we study SUSY breaking terms in the framework of $S_4 \times Z_4 \times U(1)_{FN}$ to derive squark and slepton mass matrices. We consider the gravity mediation within the framework of the supergravity theory. We assume that non-vanishing F-terms of gauge and flavor singlet (moduli) fields Z and gauge singlet fields χ_i ($i = 1, \cdots, 14$) contribute to the SUSY breaking. Their F-components are written as

$$F^{\Phi_k} = -e^{\frac{K}{2M_p^2}} K^{\Phi_k \bar{I}}\left(\partial_{\bar{I}}\bar{W} + \frac{K_{\bar{I}}}{M_p^2}\bar{W}\right), \tag{41}$$

where M_p is the Planck mass, W is the superpotential, K denotes the Kähler potential, $K_{\bar{I}J}$ denotes second derivatives by fields, i.e. $K_{\bar{I}J} = \partial_{\bar{I}}\partial_J K$ and $K^{\bar{I}J}$ is its inverse. Here the fields Φ_k correspond to the moduli fields Z and gauge singlet fields χ_i. The VEV's of F_{Φ_k}/Φ_k are estimated as $\langle F_{\Phi_k}/\Phi_k\rangle = \mathcal{O}(m_{3/2})$, where $m_{3/2}$ denotes the gravitino mass, which is obtained as $m_{3/2} = \langle e^{K/2M_p^2} W/M_p^2\rangle$.

First, let us study soft scalar masses. Within the framework of the supergravity theory, the soft scalar mass squared is obtained as [168]

$$m^2_{\bar{I}J} K_{\bar{I}J} = m^2_{3/2} K_{\bar{I}J} + |F^{\Phi_k}|^2 \partial_{\Phi_k} \partial_{\bar{\Phi}_k} K_{\bar{I}J} - |F^{\Phi_k}|^2 \partial_{\bar{\Phi}_k} K_{\bar{I}L} \partial_{\Phi_k} K_{\bar{M}J} K^{L\bar{M}}. \tag{42}$$

The invariance under the $S_4 \times Z_4 \times U(1)_{FN}$ flavor symmetry as well as the gauge invariance requires the following form of the Kähler potential as

$$K = Z^{(5)}(\Phi) \sum_{i=1,2,3} |F_i|^2 + Z^{(10)}_{(1)}(\Phi) \sum_{i=1,2} |T_i|^2 + Z^{(10)}_{(2)}(\Phi) |R_\tau|^2, \tag{43}$$

at the lowest level, where $Z^{(5)}(\Phi)$ and $Z^{(10)}_{(1),(2)}(\Phi)$ are arbitrary functions of the singlet fields Φ. By use of Eq. (42) with the Kähler potential in Eq. (43), we obtain the following matrix form of soft scalar masses squared for $\bar{5}\,\bar{5}^c$ and $10\ 10^c$ combinations, which are denoted as m_F^2 and m_T^2, respectively:

$$(m_F^2)_{ij} = \begin{pmatrix} m_F^2 & 0 & 0 \\ 0 & m_F^2 & 0 \\ 0 & 0 & m_F^2 \end{pmatrix}, \qquad (m_T^2)_{ij} = \begin{pmatrix} m^2_{T(1)} & 0 & 0 \\ 0 & m^2_{T(1)} & 0 \\ 0 & 0 & m^2_{T(2)} \end{pmatrix}. \tag{44}$$

That is, three right-handed down-type squark and left-handed slepton masses are degenerate, and first two generations of other sectors are degenerate. These predictions would be obvious because the three generations of the $F \subset (d^c, L)$ fields form a triplet of S_4, and the $T \subset (Q, u^c, e^c)$ fields form a doublet and a singlet of S_4. These predictions hold exactly before $S_4 \times Z_4 \times U(1)_{FN}$ is broken, but its breaking gives next-to-leading terms in the scalar mass matrices.

Next, we study effects due to $S_4 \times Z_4 \times U(1)_{FN}$ breaking by χ_i. That is, we estimate corrections to the Kähler potential including χ_i. Since (T_1, T_2) are assigned to **2** and its conjugate representation is itself **2**. Similarly, (F_1, F_2, F_3) are assigned to **3** and its conjugation is **3**. Therefore, for the $F_{1,2,3}$ fields, higher dimensional terms are given as

$$\begin{aligned}
\Delta K_F = &\sum_{i=1,3} Z^{(F)}_{\Delta_{a_i}}(\Phi)(F_1, F_2, F_3) \otimes (F_1^c, F_2^c, F_3^c) \otimes (\chi_i, \chi_{i+1}) \otimes (\chi_i^c, \chi_{i+1}^c)/\Lambda^2 \\
&+ \sum_{i=5,8,11} Z^{(F)}_{\Delta_{b_i}}(\Phi)(F_1, F_2, F_3) \otimes (F_1^c, F_2^c, F_3^c) \otimes (\chi_i, \chi_{i+1}, \chi_{i+2}) \otimes (\chi_i^c, \chi_{i+1}^c, \chi_{i+2}^c)/\Lambda^2 \\
&+ Z^{(F)}_{\Delta_c}(\Phi)(F_1, F_2, F_3) \otimes (F_1^c, F_2^c, F_3^c) \otimes \chi_{14} \otimes \chi_{14}^c/\Lambda^2 \\
&+ Z^{(F)}_{\Delta_d}(\Phi)(F_1, F_2, F_3) \otimes (F_1^c, F_2^c, F_3^c) \otimes \Theta \otimes \Theta^c/\bar{\Lambda}^2.
\end{aligned} \tag{45}$$

For example, higher dimensional terms including (χ_1, χ_2) and (χ_5, χ_6, χ_7) are explicitly written as

$$\begin{aligned}
\Delta K_F^{[\chi_1, \chi_5]} = &Z^{(F)}_{\Delta_{a_1}}(\Phi) \left[\frac{\sqrt{2}|\chi_1|^2}{\Lambda^2} (|F_2|^2 - |F_3|^2) \right] \\
&+ Z^{(F)}_{\Delta_{b_5}}(\Phi) \left[\frac{2|\chi_5|^2}{\Lambda^2} (F_2 F_3^* + F_3 F_2^* + F_1 F_3^* + F_3 F_1^* + F_1 F_2^* + F_2 F_1^*) \right].
\end{aligned} \tag{46}$$

When we take into account corrections from all $\chi_i\chi_j^*$ to the Kähler potential, the soft scalar masses squared for the $F_{1,2,3}$ fields have the following corrections,

$$(m_F^2)_{ij} = \begin{pmatrix} m_F^2 + \tilde{a}_{F1}^2 m_{3/2}^2 & k_F a_5^2 m_{3/2}^2 & k_F a_5^2 m_{3/2}^2 \\ k_F a_5^2 m_{3/2}^2 & m_F^2 + \tilde{a}_{F2}^2 m_{3/2}^2 & k_F a_5^2 m_{3/2}^2 \\ k_F a_5^2 m_{3/2}^2 & k_F a_5^2 m_{3/2}^2 & m_F^2 + \tilde{a}_{F3}^2 m_{3/2}^2 \end{pmatrix}, \tag{47}$$

where k_F is a parameter of order one, and $\tilde{a}_{Fk}^2 (k = 1, 2, 3)$ are linear combinations of $a_i a_j$'s.

For the $T_{1,2,3}$ fields, higher dimensional terms are given as

$$\begin{aligned} \Delta K_T = &\sum_{i=1,3} Z_{\Delta_{a_i}}^{(T)}(\Phi)(T_1,T_2)\otimes(T_1^c,T_2^c)\otimes(\chi_i,\chi_{i+1})\otimes(\chi_i^c,\chi_{i+1}^c)/\Lambda^2 \\ &+ \sum_{i=5,8,11} Z_{\Delta_{b_i}}^{(T)}(\Phi)(T_1,T_2)\otimes(T_1^c,T_2^c)\otimes(\chi_i,\chi_{i+1},\chi_{i+2})\otimes(\chi_i^c,\chi_{i+1}^c,\chi_{i+2}^c)/\Lambda^2 \\ &+ Z_{\Delta_c}^{(T)}(\Phi)(T_1,T_2)\otimes(T_1^c,T_2^c)\otimes\chi_{14}\otimes\chi_{14}^c/\Lambda^2 \\ &+ Z_{\Delta_d}^{(T)}(\Phi)(T_1,T_2)\otimes T_3^c\otimes(\chi_1,\chi_2)/\Lambda^2 + Z_{\Delta_e}^{(T)}(\Phi)(T_1^c,T_2^c)\otimes T_3\otimes(\chi_1^c,\chi_2^c)/\Lambda^2 \\ &+ \sum_{i=1,3} Z_{\Delta_{f_i}}^{(T)}(\Phi)T_3\otimes T_3^c\otimes(\chi_i,\chi_{i+1})\otimes(\chi_i^c,\chi_{i+1}^c)/\Lambda^2 \\ &+ \sum_{i=5,8,11} Z_{\Delta_{g_i}}^{(T)}(\Phi)T_3\otimes T_3^c\otimes(\chi_i,\chi_{i+1},\chi_{i+2})\otimes(\chi_i^c,\chi_{i+1}^c,\chi_{i+2}^c)/\Lambda^2 \\ &+ Z_{\Delta_h}^{(T)}(\Phi)T_3\otimes T_3^c\otimes\chi_{14}\otimes\chi_{14}^c/\Lambda^2 + Z_{\Delta_i}^{(T)}(\Phi)(T_1,T_2)\otimes(T_1^c,T_2^c)\otimes\Theta\otimes\Theta^c/\bar{\Lambda}^2 \\ &+ Z_{\Delta_j}^{(T)}(\Phi)T_3\otimes T_3^c\otimes\Theta\otimes\Theta^c/\bar{\Lambda}^2. \end{aligned} \tag{48}$$

In the same way, the $T_{1,2,3}$ scalar mass matrix can be written as

$$(m_T^2)_{ij} = \begin{pmatrix} m_{T(1)}^2 + \tilde{a}_{T11}^2 m_{3/2}^2 & \tilde{a}_{T12}^2 m_{3/2}^2 & k_T a_1 m_{3/2}^2 \\ \tilde{a}_{T12}^2 m_{3/2}^2 & m_{T(1)}^2 + \tilde{a}_{T22}^2 m_{3/2}^2 & k_T a_1 m_{3/2}^2 \\ k_T^* a_1 m_{3/2}^2 & k_T^* a_1 m_{3/2}^2 & m_{T(2)}^2 + \tilde{a}_{T33}^2 m_{3/2}^2 \end{pmatrix}, \tag{49}$$

where k_T is a complex parameter whose magnitude is of order one, and it is the only new source of the CP violation in our model. The parameters $\tilde{a}_{Tij}^2$ are linear combinations of $a_k a_\ell$'s. In numerical analysis, we use the parameter Δa_L which is given by $\Delta a_L = m_{T(2)}^2/m_{T(1)}^2 - 1$.

In order to estimate the magnitude of FCNC phenomena, we move to the super-CKM basis by diagonalizing quark and lepton mass matrices including next-to-leading terms. For the left-handed down-type squark and slepton, we get

$$(\tilde{m}_{d_{LL}}^2)_{ij}^{(SCKM)} = U_d^\dagger (m_T^2)_{ij} U_d, \quad (\tilde{m}_{\ell_{LL}}^2)_{ij}^{(SCKM)} = U_E^\dagger (m_F^2)_{ij} U_E, \tag{50}$$

and for the right-handed down-type squark and slepton as

$$(\tilde{m}_{d_{RR}}^2)_{ij}^{(SCKM)} = V_d^\dagger (m_F^2)_{ij} V_d, \quad (\tilde{m}_{e_{RR}}^2)_{ij}^{(SCKM)} = V_E^\dagger (m_T^2)_{ij} V_E, \tag{51}$$

where the mixing matrices V_E and U_E are given in Eq. (C11) in Appendix C.

Let us study scalar trilinear couplings, i.e. the so called A-terms. The A-terms among left-handed and right-handed squarks (sleptons) and Higgs scalar fields are obtained in the gravity mediation as [168]

$$h_{IJ}L_J R_I H_K = \sum_{K=\bar{5},\ 45} h^{(Y)}_{IJK} L_J R_I H_K + h^{(K)}_{IJK} L_J R_I H_K, \tag{52}$$

where

$$\begin{aligned} h^{(Y)}_{IJK} &= F^{\Phi_k}\langle \partial_{\Phi_k}\tilde{y}_{IJK}\rangle, \\ h^{(K)}_{IJK} L_J R_I H_K &= -\langle \tilde{y}_{LJK}\rangle L_J R_I H_K F^{\Phi_k} K^{L\bar{L}} \partial_{\Phi_k} K_{\bar{L}I} \\ &\quad -\langle \tilde{y}_{IMK}\rangle L_J R_I H_d F^{\Phi_k} K^{M\bar{M}} \partial_{\Phi_k} K_{\bar{M}J} \\ &\quad -\langle \tilde{y}_{IJK}\rangle L_J R_I H_K F^{\Phi_k} K^{H_d} \partial_{\Phi_k} K_{H_K}, \end{aligned} \tag{53}$$

and K_{H_K} denotes the Kähler metric of H_K. In addition, effective Yukawa couplings of the down-type quark $\tilde{y}_{IJK}$ are written as

$$\tilde{y}_{IJK} = y_1 \begin{pmatrix} 0 & a_9/\sqrt{2} & 0 \\ 0 & a_9/\sqrt{6} & 0 \\ 0 & 0 & 0 \end{pmatrix}_{LR} + y_2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_{13} \end{pmatrix}_{LR}, \tag{54}$$

then we have

$$h^{(Y)}_{IJK} = \frac{y_1}{\Lambda} \begin{pmatrix} 0 & \tilde{F}^{a_9}/\sqrt{2} & 0 \\ 0 & \tilde{F}^{a_9}/\sqrt{6} & 0 \\ 0 & 0 & 0 \end{pmatrix}_{LR} + \frac{y_2}{\Lambda} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \tilde{F}^{a_{13}} \end{pmatrix}_{LR}, \tag{55}$$

where $\tilde{F}^{a_i} = F^{a_i}/a_i$ and $\tilde{F}^{a_i}/\Lambda = \mathcal{O}(m_{3/2})$.

By use of lowest level of the Kähler potential, we estimate $h^{(K)}_{IJK}$ as

$$h^{(K)}_{IJK} = \tilde{y}_{IJK}(A^R_I + A^L_J), \tag{56}$$

where we assume $A^L_1 = A^L_2 = A^L_3 = F^{\tilde{a}_i}/(a_i\Lambda) \simeq \mathcal{O}(m_{3/2})$. The magnitudes of $A^R_1 = A^R_2$ and A^R_3 are also $\mathcal{O}(m_{3/2})$. Furthermore, we should take into account next-to-leading terms of the Kähler potential including χ_i. These correction terms appear all entries so that their magnitudes are suppressed in $\mathcal{O}(\tilde{a})$ compared with the leading term. Then, we obtain

$$(m^2_{d_{LR}})_{ij} \simeq (m^2_{\ell_{LR}})^\dagger_{ij} \simeq m_{3/2} \begin{pmatrix} \tilde{a}^2_{LR11} v_d & c_1 \frac{\sqrt{3} m_{s(\mu)}}{2} & \tilde{a}^2_{LR13} v_d \\ \tilde{a}^2_{LR21} v_d & c_1 \frac{m_{s(\mu)}}{2} & \tilde{a}^2_{LR23} v_d \\ \tilde{a}^2_{LR31} v_d & \tilde{a}^2_{LR32} v_d & c_2 m_{b(\tau)} \end{pmatrix}_{LR}, \tag{57}$$

where $\tilde{a}^2_{LRij}$ are linear combinations of $a_k a_\ell$'s, and c_1 and c_2 are of order one parameters. Moving to the super-CKM basis, we have

$$(\tilde{m}^2_{d_{LR}})^{(SCKM)}_{ij} = U^\dagger_d (m^2_{d_{LR}})_{ij} V_d \simeq m_{3/2} \begin{pmatrix} \mathcal{O}(\tilde{a}^2 v_d) & \mathcal{O}(\tilde{a}^2 v_d) & \mathcal{O}(\tilde{a}^2 v_d) \\ \mathcal{O}(\tilde{a}^2 v_d) & \mathcal{O}(m_s) & \mathcal{O}(\tilde{a}^2 v_d) \\ \mathcal{O}(\tilde{a}^2 v_d) & \mathcal{O}(\tilde{a}^2 v_d) & \mathcal{O}(m_b) \end{pmatrix}. \tag{58}$$

Similarly, for the charged lepton,

$$(\tilde{m}^2_{\ell_{LR}})^{(SCKM)}_{ij} = U_E^\dagger (m^2_{\ell_{LR}})_{ij} V_E \simeq m_{3/2} \begin{pmatrix} \mathcal{O}\left(\tilde{a}^2 v_d\right) & \mathcal{O}\left(\tilde{a}^2 v_d\right) & \mathcal{O}\left(\tilde{a}^2 v_d\right) \\ \mathcal{O}\left(\tilde{a}^2 v_d\right) & \mathcal{O}(m_\mu) & \mathcal{O}\left(\tilde{a}^2 v_d\right) \\ \mathcal{O}\left(\tilde{a}^2 v_d\right) & \mathcal{O}\left(\tilde{a}^2 v_d\right) & \mathcal{O}(m_\tau) \end{pmatrix} . \tag{59}$$

C. Renormalization group effect

In the framework of the supergravity, soft masses for all scalar particles have the common scale denoted by $m_{\rm SUSY}$, and gauginos also have the common scale $m_{1/2}$. Therefore, at the GUT scale $m_{\rm GUT}$, we take

$$M_1(m_{\rm GUT}) = M_2(m_{\rm GUT}) = M_3(m_{\rm GUT}) = m_{1/2} . \tag{60}$$

Effects of the renormalization group running lead at the scale m_W to following masses for gauginos,

$$M_i(m_W) \simeq \frac{\alpha_i(m_W)}{\alpha_i(m_{\rm GUT})} M_i(m_{\rm GUT}). \tag{61}$$

Taking into account the renormalization group effect [169] on the average mass scale in $m^2_{e_L}$, $m^2_{e_R}$, $m^2_{q_L}$, and $m^2_{d_R}$ with neglecting Yukawa couplings, we have

$$\begin{aligned} m^2_{e_L}(m_W) &\simeq m^2_L(m_{\rm GUT}) + 0.5 M_2^2(m_{\rm GUT}) + 0.04 M_1^2(m_{\rm GUT}) \simeq m^2_{\rm SUSY} + 0.54 m^2_{1/2}, \\ m^2_{e_R}(m_W) &\simeq m^2_R(m_{\rm GUT}) + 0.15 M_1^2(m_{\rm GUT}) \simeq m^2_{\rm SUSY} + 0.15 m^2_{1/2} , \\ m^2_{q_L}(m_W) &\simeq m^2_R(m_{\rm GUT}) + 0.004 M_1^2(m_{\rm GUT}) + 0.4 M_2^2(m_{\rm GUT}) + 3.6 M_3^2(m_{\rm GUT}) \\ &\simeq m^2_{\rm SUSY} + 4.1 m^2_{1/2} , \\ m^2_{d_R}(m_W) &\simeq m^2_R(m_{\rm GUT}) + 0.015 M_1^2(m_{\rm GUT}) + 3.6 M_3^2(m_{\rm GUT}) \simeq m^2_{\rm SUSY} + 3.7 m^2_{1/2} . \end{aligned} \tag{62}$$

For Yukawa couplings, the $b-\tau$ unification is realized at the leading order in our model, however, the $b-\tau$ unification is deviated when we include the next-to-leading order mass operators due to terms including H_{45}, see Ref. [147] for the detail. In that paper, we have calculated the renormalization group equations and observed fermion masses at the weak scale can be obtained when $\tan\beta$ is larger than two. Hereafter, we take $\tan\beta = 3$ on the numerical analysis.

IV. NUMERICAL ANALYSIS

In this section, we perform numerical analysis to show that the S_4 flavor model presented in the previous section can explain the like-sign charge asymmetry in the $B_s - \bar{B}_s$ system. In order for this calculation, we first define the MI parameters for down-type squarks δ_d^{LL}, δ_d^{LR}, δ_d^{RL}, and δ_d^{RR} and for sleptons δ_ℓ^{LL}, δ_ℓ^{LR}, δ_ℓ^{RL} and, δ_e^{RR} as

$$\begin{aligned} m^2_{\tilde{q}} \begin{pmatrix} \delta_d^{LL} & \delta_d^{LR} \\ \delta_d^{RL} & \delta_d^{RR} \end{pmatrix} &= \begin{pmatrix} (\tilde{m}^2_{d_{LL}})^{(SCKM)} & (\tilde{m}^2_{d_{LR}})^{(SCKM)} \\ (\tilde{m}^2_{d_{RL}})^{(SCKM)} & (\tilde{m}^2_{d_{RR}})^{(SCKM)} \end{pmatrix} - \mathrm{diag}(m^2_{\tilde{q}}) , \\ m^2_{\tilde{\ell}} \begin{pmatrix} \delta_\ell^{LL} & \delta_\ell^{LR} \\ \delta_\ell^{RL} & \delta_e^{RR} \end{pmatrix} &= \begin{pmatrix} (\tilde{m}^2_{\ell_{LL}})^{(SCKM)} & (\tilde{m}^2_{\ell_{LR}})^{(SCKM)} \\ (\tilde{m}^2_{\ell_{RL}})^{(SCKM)} & (\tilde{m}^2_{e_{RR}})^{(SCKM)} \end{pmatrix} - \mathrm{diag}(m^2_{\tilde{\ell}}) , \end{aligned} \tag{63}$$

where $m_{\tilde{q}}$ and $m_{\tilde{\ell}}$ are average squark and slepton masses with the values given below.

In the numerical analysis, we fix the following parameters

$$\begin{aligned} m_{3/2} &= 430 \text{ GeV},\ m_{\tilde{q}} = 880 \text{ GeV},\ m_{\tilde{\ell}} = 520 \text{ GeV}, \\ M_1 &= 135 \text{ GeV},\ M_2 = 270 \text{ GeV},\ M_3 \equiv m_{\tilde{g}} = 1 \text{ TeV}, \end{aligned} \tag{64}$$

which are derived from the universal relation at the GUT scale

$$m_{1/2}(m_{\text{GUT}}) = m_{3/2}(m_{\text{GUT}}) = m_{\text{SUSY}}(m_{\text{GUT}}) = 430 \text{ GeV}, \tag{65}$$

by through the renormalization group effect discussed in the section III-C. This universal value is taken to be consistent with the lower bound of the gluino mass, which has been reported recently at Atlas Collaboration of LHC [159–161]. For the other parameters, we assume the following regions:

$$\begin{aligned} \mu &= [500, 1000] \text{ GeV},\ \Delta a_L = [-0.5, 5],\ |k_T a_1| = [0, 2],\ \arg(k_T a_1) = [-\pi, \pi], \\ a_5 &= [0, 0.001],\ \tilde{a}_{T12} = [0, 0.1],\ \tilde{a}_{LRij} = [0, 0.01], \end{aligned} \tag{66}$$

with $\tan\beta = 3$, $c_{1,2} = 1$, and $k_F = 1$. In Eq.(66), the number of left-handed and right-handed sides in braces denote the minimal and maximal values, respectively. In our calculation, we neglect the diagonal elements of scalar masses $\tilde{a}_{F(1,2,3)}$ and $\tilde{a}_{T(11,22,33)}$. The leading contribution to the parameters $\tilde{a}$ in the soft-terms are a_1 as $\tilde{a}_{LRij} \simeq \sqrt{a_1 a_5}$ and $\tilde{a}_{T12} \simeq a_1$. As given in Appendix D, the SUSY contribution to $M_{12}^{s,SUSY}$ is estimated as

$$\begin{aligned} M_{12}^{s,SUSY} \simeq -\frac{\alpha_S^2}{216 m_{\tilde{q}}^2}\frac{2}{3}M_{B_s} f_{B_s}^2 \Big\{ &-0.59\left[(\delta_d^{LL})_{23}^2 + (\delta_d^{RR})_{23}^2\right] + 31(\delta_d^{LL})_{23}(\delta_d^{RR})_{23} \\ &-9.4\left[(\delta_d^{LR})_{23}^2 + (\delta_d^{RL})_{23}^2\right] + 7.9(\delta_d^{LR})_{23}(\delta_d^{RL})_{23}\Big\}, \end{aligned} \tag{67}$$

for $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2 \simeq 1.3$, and similar for B_d mixing. The coefficients in front of MI parameters for K meson mixing are $-0.59, 554, -183, 114$, respectively. Since the LR terms $(\delta_d^{LR,RL})_{ij}$ are strictly constrained by $b \to s\gamma$ as seen in Appendix D, the $(\delta_d^{LL(RR)})_{ij}$ terms gives larger contribution to $M_{12}^{s,SUSY}$. Among them, since $(\delta_d^{RR})_{ij} \simeq k_F a_5^2 \lesssim 10^{-6}$ in our parameter region given in Eq.(66), the first term $(\delta_d^{LL})_{ij}^2$ gives the dominant contributions. The approximation form of the LL parameters $(\delta_d^{LL})_{ij}$ are given by

$$\begin{aligned} (\delta_d^{LL})_{12} &\simeq \theta_{13}^d\theta_{23}^d\Delta a_L - \frac{m_{3/2}^2}{m_{\tilde{q}}^2}\left(\theta_{13}^d\frac{1+\sqrt{3}}{2}k_T^* a_1 + \theta_{23}^d\frac{1-\sqrt{3}}{2}k_T a_1\right) \\ &\simeq -\frac{m_{3/2}^2}{m_{\tilde{q}}^2}\sqrt{2}\left(\theta_{13}^d V_{ud} k_T^* a_1 - \theta_{23}^d V_{us} k_T a_1\right), \end{aligned} \tag{68}$$

$$(\delta_d^{LL})_{13} \simeq -\theta_{13}^d\Delta a_L + \frac{m_{3/2}^2}{m_{\tilde{q}}^2}\left(\frac{1-\sqrt{3}}{2} - \theta_{12}^d\frac{1+\sqrt{3}}{2}\right)k_T a_1 \simeq -\frac{m_{3/2}^2}{m_{\tilde{q}}^2}\sqrt{2}V_{us}k_T a_1, \tag{69}$$

$$(\delta_d^{LL})_{23} \simeq -\theta_{23}^d\Delta a_L + \frac{m_{3/2}^2}{m_{\tilde{q}}^2}\left(\frac{1+\sqrt{3}}{2} + \theta_{12}^d\frac{1-\sqrt{3}}{2}\right)k_T a_1 \simeq \frac{m_{3/2}^2}{m_{\tilde{q}}^2}\sqrt{2}V_{ud}k_T a_1, \tag{70}$$

where in the last approximation of each expression, we have neglected the first term proportional to $\theta^d_{13,23} \simeq 0.005$ and $\Delta a_L \sim 1$. Notice that the MI parameters are expressed in terms of the CKM elements, and that both $(\delta_d^{LL})_{13}$ and $(\delta_d^{LL})_{23}$ have the same phase structure $k_T a_1$, which is only the new source of the CP violation in our model. These are the typical feature of our S_4 flavor model. By using these expressions, one finds that the $K^0 - \bar{K}^0$ mixing induced by $(\delta_d^{LL})_{12}$ is more suppressed by additional factor θ^d_{ij}.

The cEDM for the strange quark is estimated from the formula in Appendix D as

$$ed_s^C \sim 10^{-20}\mathrm{Im}[(\delta_d^{LL})_{23}(\delta_d^{LR})_{33}(\delta_d^{RR})_{32}] \; e\mathrm{cm} \sim 10^{-28}\mathrm{Im}[(\delta_d^{LL})_{23}] \; e\mathrm{cm}, \tag{71}$$

for $x \simeq 1.3$, $(\delta^d_{33})_{LR} \sim 10^{-2}$ and $(\delta^d_{32})_{RR} \sim 10^{-6}$. Therefore, $(\delta^d_{23})_{LL}$ is not constrained by cEDM. As for the $b \to s\gamma$ process, one can see that $(\delta^d_{23})_{LR}$ should be strongly suppressed while $(\delta^d_{23})_{LL,RR}$ have an additional suppression factor $m_b/m_{\tilde{g}} \sim 10^{-3}$. In our numerical calculation, we take $\tilde{a}_{LRij} \lesssim 0.01$ so that $b \to s\gamma$ is well suppressed, and the allowed region of $|(\delta_d^{LL})_{23}|$ is also small enough as mentioned below.

First we discuss the allowed regions of the parameters (h_s, h_d), (h_s, σ_s), (h_d, σ_d) defined in Eq.(11). In our model, the parameters $h_{d,s} e^{2i\sigma_{d,s}}$ are estimated as

$$h_d e^{2i\sigma_d} = \frac{M_{12}^{d,SUSY}}{M_{12}^{d,SM}} \simeq (27 - i25)(\delta_d^{LL})^2_{13}, \tag{72}$$

$$h_s e^{2i\sigma_s} = \frac{M_{12}^{s,SUSY}}{M_{12}^{s,SM}} \simeq (1.7 + i0.06)(\delta_d^{LL})^2_{23}, \tag{73}$$

where the MI parameters $(\delta_d^{LL})_{ij}$ reflect the flavor symmetry, while the factors $(27 - i25)$ and $(1.7 + i0.06)$ do not. The ratio of $(27-i25)/(1.7+i0.06)$ is related to the CKM elements as $(27-i25)/(1.7+i0.06) \simeq M_{12}^{s,SM}/M_{12}^{d,SM} \simeq (V^*_{ts}/V^*_{td})^2$. We obtain the ratio of h_d and h_s as

$$\frac{h_d}{h_s} \simeq \frac{|27 - i25|}{|1.7 + i0.06|}\frac{|V_{us}|^2}{|V_{ud}|^2} \simeq \frac{|V_{ts}|^2}{|V_{td}|^2}\frac{|V_{us}|^2}{|V_{ud}|^2} \simeq 1. \tag{74}$$

Therefore, the fact that the region $h_d \simeq h_s$ is favored reflects the flavor structure of the S_4 flavor model. The CP violation phase ϕ_s is given by

$$\phi_s \simeq \arg\left[-(1 + h_s e^{2i\sigma_s})\right], \tag{75}$$

with neglecting the SM contribution. The CP phase $\sin\phi_s$ is bounded as $|\sin\phi_s| \lesssim h_s$ for $h_s < 1$, and has the negatively-maximal value $\sin\phi_s \simeq -h_s$ at $\sigma_s \simeq 120°$. This corresponds to the best-fit value $(h_s, \sigma_s) = (0.5, 120°)$ of Eq.(16)[162].

Fig.1 shows the plot in the $\phi_s - A^b_{sl}$ plane. The horizontal and vertical lines are the experimental values of one-dimensional likelihood analysis [170]

$$\phi_s = [-1.8, 0.4] \text{ (rad)}, \text{ at } 95\% \text{ C.L.}, \tag{76}$$

and 2σ range of A^b_{sl} in Eq.(8), respectively. The blue (dark gray) and orange (light gray) regions denote 2σ and 3σ regions of A^b_{sl} in Eq.(8), respectively. By using Eq.(14) and $|1 + h_s \exp(2i\sigma_s)| \sim$

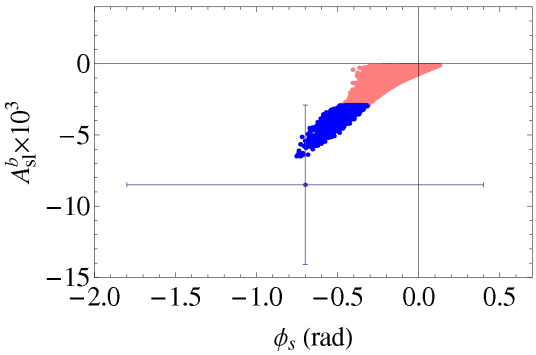

FIG. 1: Allowed region in the $\phi_s - A^b_{sl}$ plane. The blue (dark gray) and orange (light gray) regions denote 2σ and 3σ regions of A^b_{sl} in Eq. (8). The blue (black) error bars of the horizontal and vertical lines are experimental values of 2σ region of ϕ_s and A^b_{sl}.

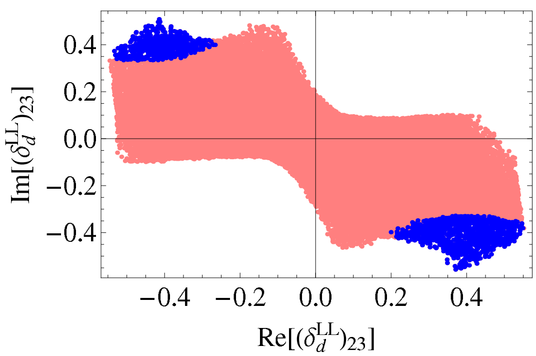

FIG. 2: Allowed region in the $\mathrm{Re}(\delta_d^{LL})_{23} - \mathrm{Im}(\delta_d^{LL})_{23}$ plane. The blue (dark gray) and orange (light gray) regions denote 2σ and 3σ regions of A^b_{sl} in Eq. (8).

$(1 + 0.5\exp[2i120°]) \simeq 0.8$, we obtain $-a^s_{sl} \gtrsim 10^{-3}$, and similar for a^d_{sl}. As a consequence we obtain the like-sign charge asymmetry as $-A^b_{sl} \gtrsim 10^{-3}$, which is within 2σ range of A^b_{sl} of Eq.(8).

Fig.2 shows the allowed region in the $\mathrm{Re}(\delta_d^{LL})_{23} - \mathrm{Im}(\delta_d^{LL})_{23}$ plane. One finds from Eq.(73) that in order to obtain the best fit value $(h_s, \sigma_s) = (0.5, 120°)$, the sign of $\mathrm{Re}(\delta_d^{LL})_{23}$ and $\mathrm{Im}(\delta_d^{LL})_{23}$ must be opposite from each other, with $\mathrm{Re}(\delta_d^{LL})_{23} \simeq \pm 0.3$ and $\mathrm{Im}(\delta_d^{LL})_{23} \simeq \mp 0.4$. This allowed region $|(\delta_d^{LL})_{23}| \lesssim 0.5$ is small enough to suppress $b \to s\gamma$. The similar figure is drawn in the $\mathrm{Re}(\delta_d^{LL})_{13} - \mathrm{Im}(\delta_d^{LL})_{13}$ plane with $|V_{td}/V_{ts}| \simeq 0.22$ times smaller area.

In Fig.3, we predict the difference $a^s_{sl} - a^d_{sl}$, which will be measured at LHCb, as a function of ϕ_s. The SM prediction [151] $a^{s,SM}_{sl} - a^{d,SM}_{sl} = (4.3 \pm 0.7) \times 10^{-4}$ is also shown. We predict that $a^s_{sl} - a^d_{sl} \simeq (1 - 5) \times 10^{-3}$ in the 2σ region of A^b_{sl}. This will be a good test for our S_4 flavor model.

Since our model is based on the $SU(5)$ GUT, above contributions in the quark sector affect to the lepton sector. Therefore, sleptons contribute to the LFV processes and EDM of the electron [171, 172], in which the experimental measurements give the upper bounds [173–175]. The Fig.4 shows the relation of $\mathrm{BR}(\mu \to e\gamma)$ and the electron EDM. Within the MI parameters, $(\delta_e^{RR})_{ij} \sim (\delta_d^{LL})_{ji}$ except for $(i, j) = (1, 2), (2, 1)$ are relatively large in our model. Therefore one finds from Appendix E that the $(\delta_e^{RL})_{21}$ term in A^{21}_L, which is enhanced by $M_1/m_\mu \sim 10^3$, mainly contributes to $\mu \to e\gamma$

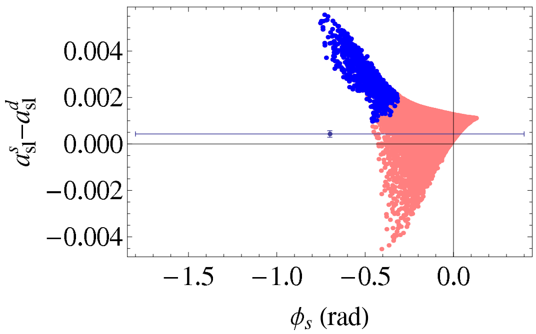

FIG. 3: Allowed region in the $(a^s_{sl} - a^d_{sl}) - \phi_s$ plane. The blue (dark gray) and orange (light gray) regions denote 2σ and 3σ regions of A^b_{sl} in Eq. (8). The blue error bars of the horizontal and vertical lines are experimental values of 2σ region of ϕ_s and $(a^s_{sl} - a^d_{sl})$. The horizontal line is experimental values of 2σ region for ϕ_s, and the vertical line is the SM prediction of $(a^s_{\rm sl} - a^d_{\rm sl})$.

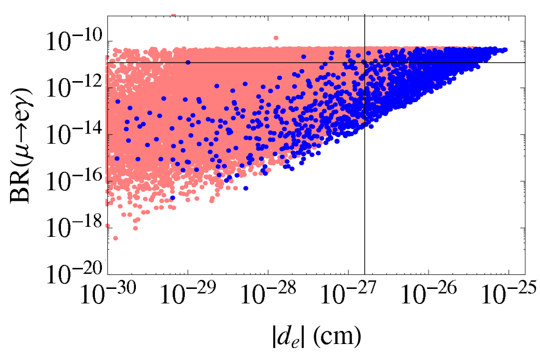

FIG. 4: Allowed region in the $|d_e| - BR(\mu \to e\gamma)$ plane. The blue (dark gray) and orange (light gray) regions denote 2σ and 3σ regions of A^b_{sl} in Eq. (8). The horizontal and vertical lines are experimental bounds of $|d_e|$ and $BR(\mu \to e\gamma)$.

process. As for the electron EDM, the terms with one small MI parameters dominates. The largest contributions are approximately estimated as

$$\begin{aligned}
\mathrm{BR}(\mu \to e\gamma) &\simeq \frac{48\pi^3\alpha}{G_F^2}\left|\frac{\alpha_1}{4\pi}\frac{(\delta_e^{RL})_{21}}{m_{\tilde{\ell}}^2}\left(\frac{M_1}{m_\mu}\right)2f_{2n}(x_1)\right|^2 \\
&\simeq 3\times 10^{-11}\left(\frac{520\mathrm{GeV}}{m_{\tilde{\ell}}}\right)^4\left(\frac{M_1}{135\mathrm{GeV}}\right)^2\left(\frac{|(\delta_e^{RL})_{21}|}{10^{-5}}\right)^2, \qquad (77)\\
|d_e/e| &\simeq \frac{\alpha_1}{4\pi}\frac{M_1}{m_{\tilde{\ell}}^2}\Big|\mathrm{Im}[(\delta_\ell^{LR})_{13}(\delta_e^{RR})_{31}]f_{3n}(x_1) \\
&+ \mathrm{Im}[(\delta_\ell^{LR})_{12}(\delta_e^{RR})_{23}(\delta_e^{RR})_{31} + (\delta_\ell^{LR})_{13}(\delta_e^{RR})_{33}(\delta_e^{RR})_{31}]f_{4n}(x_1)\Big| \\
&\simeq 1\times 10^{-26}cm\times\left(\frac{M_1}{135\mathrm{GeV}}\right)\left(\frac{520\mathrm{GeV}}{m_{\tilde{\ell}}}\right)^2\left[\left(\frac{(\delta_\ell^{LR})_{13}}{10^{-5}}\right)\left(\frac{(\delta_e^{RR})_{31}}{0.1}\right)+\cdots\right]. \qquad (78)
\end{aligned}$$

The value of $|(\delta_e^{RL})_{ij}|$ is of order $m_{3/2}v_d/m_{\tilde{\ell}}^2 \times \tilde{a}_{LRij}^2 \lesssim 10^{-5}$. Therefore we find that BR($\mu \to e\gamma$) and electron EDM can be close to the present experimental bound as shown in the figure.

The $b-s$ transition by $(\delta_d^{LL})_{23}$ in the quark sector simultaneously induce the LFV τ decay $\tau \to \mu\gamma$ by $(\delta_e^{RR})_{32}$. The dominant contribution is estimated from Appendix E as

$$\begin{aligned}\mathrm{BR}(\tau \to \mu\gamma) &\simeq \mathrm{BR}(\tau \to \mu\nu_\tau\bar{\nu}_\mu)\frac{48\pi^3\alpha}{G_F^2}\left(\frac{\alpha_1}{4\pi}\right)^2\left[\frac{(\delta_e^{RR})_{32}}{m_{\tilde{\ell}}^2}\mu M_1 \tan\beta\left(\frac{f_{3n}(x_1)}{m_{\tilde{\ell}}^2}-\frac{2f_{2n}(x_1,x_\mu)}{\mu^2-M_1^2}\right)\right]^2 \\ &\simeq 10^{-8}\left(\frac{520\mathrm{GeV}}{m_{\tilde{\ell}}}\right)^8\left(\frac{M_1}{135\mathrm{GeV}}\right)^2\left(\frac{\tan\beta}{3}\right)^2\left(\frac{\mu}{500\mathrm{GeV}}\right)^2|(\delta_e^{RR})_{32}|^2, \end{aligned} \tag{79}$$

and similar for $\tau \to e\gamma$ decay. Therefore for large μ term, $\tau \to \mu\gamma$ can be close to present upper bound given in Eq.(21). By using the expression Eqs.(69) and (70), we obtain the relation of $\mathrm{BR}(\tau \to \mu\gamma)$ and $\mathrm{BR}(\tau \to e\gamma)$ depending on the Cabibbo angle $\lambda_c \simeq 0.22$ as follows:

$$\frac{\mathrm{BR}(\tau \to e\gamma)}{\mathrm{BR}(\tau \to \mu\gamma)} \simeq \frac{|(\delta_e^{RR})_{31}|^2}{|(\delta_e^{RR})_{32}|^2} \simeq \frac{|(\delta_d^{LL})_{13}|^2}{|(\delta_d^{LL})_{23}|^2} \simeq \frac{|V_{us}|^2}{|V_{ud}|^2} \simeq \lambda_c^2 \simeq 0.05. \tag{80}$$

Therefore we conclude that there exist the parameter region which can explain the like-sign dimuon asymmetry A_{sl}^b in the S_4 flavor model, and in this case we predict that the LFV $\tau \to \mu\gamma$ decay can be so large that future experiments will reach, and the ratio of LFV of τ decays, $\tau \to e\gamma$ and $\tau \to \mu\gamma$, depends on the Cabibbo angle λ_c.

V. SUMMARY

Recently the DØ Collaboration reported the like-sign dimuon charge asymmetry A_{sl}^b in $b\bar{b} \to \mu^\pm\mu^\pm X$ decay processes. Their result shows 3.2σ deviation from the standard model prediction. One promising interpretation of this result is that there exist additional contribution of new physics to the CP violation in $B_s - \bar{B}_s$ mixing process. In the effective Hamiltonian of the neutral B_s meson system, there are three physical quantities $|\Gamma_{12}^s|$, $|M_{12}^s|$ and the CP phase $\phi_s = \arg(-M_{12}^s/\Gamma_{12}^s)$. In order to obtain large CP asymmetry in the neutral B_s meson system, additional contributions from new physics to at least one of these three quantities are required. Within these possibilities, one can consider new physics that the absorptive part Γ_{12}^s can be enhanced. However in general supersymmetric models, the gluino-squark box diagrams give the dominant contributions to $B_s - \bar{B}_s$ mixing, which do not affect $|\Gamma_{12}^s|$. Therefore in those models, new physics contributes to $|M_{12}^s|$ and ϕ_s.

In this paper we have considered an $SU(5)$ SUSY GUT with S_4 flavor symmetry. In this model, the Cabibbo angle, $\lambda_c \sim \sin 15°$, of the quark sector is given by a difference of 45° from up sector and 60° from down sector due to the Clebsch-Gordan coefficients at the leading order. As for the lepton sector, the tri-bimaximal form is generated in neutrino sector. These are consequences of the S_4 flavor symmetry. Since the matter multiplet $T(10)$ and $F(\bar{5})$ are embedded into $\mathbf{2}+\mathbf{1}$ and $\mathbf{3}$ of the S_4 group, respectively, the scalar masses of right-handed down-type squark and left-handed slepton are degenerated at the leading order, while those of $T_{1,2,3}$ fields are degenerated in the first two generations. Moreover for scalar mass matrix of $T_{1,2,3}$ fields, the relation $(m_T^2)_{13} = (m_T^2)_{23} \propto k_T a_1$ holds due to the S_4 symmetry. The factor $k_T a_1$ in the scalar mass matrix is assumed to be the only

additional complex parameter in our model, which is responsible for the CP violation in the neutral B_s meson system via gluino-squark box diagrams. As a consequence, the mass-insertion parameters $(\delta_d^{LL})_{13}$ and $(\delta_d^{LL})_{23}$ have approximately the structure of $V_{us}k_T a_1$ and $V_{ud}k_T a_1$, respectively.

We have shown that the like-sign charge asymmetry A_{sl}^b is in the 2σ range of the combined result of DØ and CDF measurements. Since the relation between two CP phases $\sin\phi_d \simeq \sin\phi_s$ holds due to S_4 flavor symmetry, and it can be large, we obtain large wrong-sign and like-sign asymmetry: $|a_{sl}^{d,s}| \sim |A_{sl}^b| \sim 10^{-3}$. The SUSY contributions in the quark sector affect to the lepton sector because of the $SU(5)$ GUT relation $(\delta_d^{LL})_{ij} \simeq (\delta_e^{RR})_{ji}$. In the parameter region allowed by A_{sl}^b, we have two predictions in the leptonic processes: (i) Both BR$(\mu \to e\gamma)$ and the electron EDM are close to the present upper bound. Therefore, the MEG experiment [173] will be a good test of our model. (ii) The LFV τ decays, $\tau \to \mu\gamma$ and $\tau \to e\gamma$, are related to each other via the Cabibbo angle λ_c: BR$(\tau \to e\gamma)$/BR$(\tau \to \mu\gamma) \simeq \lambda_c^2$. This is also testable at future experiments such as superKEKB.

Acknowledgments

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Appendix A: Multiplication rule of S_4

The S_4 group has 24 distinct elements and irreducible representations $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{2}$, $\mathbf{3}$, and $\mathbf{3}'$. All of the S_4 elements are written by products of the generators b_1 and d_4, which satisfy

$$(b_1)^3 = (d_4)^4 = e, \quad d_4(b_1)^2 d_4 = b_1, \quad d_4 b_1 d_4 = b_1 (d_4)^2 b_1 \ . \tag{A1}$$

These generators are represented on $\mathbf{2}$, $\mathbf{3}$ and $\mathbf{3}'$ as follows,

$$b_1 = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \quad d_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \text{on } \ \mathbf{2}, \tag{A2}$$

$$b_1 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad d_4 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \text{on } \ \mathbf{3}, \tag{A3}$$

$$b_1 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad d_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \qquad \text{on } \ \mathbf{3}'. \tag{A4}$$

The multiplication rule depends on the basis. We present the multiplication rule, which is used in

this paper:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_{\mathbf{2}} = (a_1b_1 + a_2b_2)_{\mathbf{1}} \oplus (-a_1b_2 + a_2b_1)_{\mathbf{1'}} \oplus \begin{pmatrix} a_1b_2 + a_2b_1 \\ a_1b_1 - a_2b_2 \end{pmatrix}_{\mathbf{2}} , \tag{A5}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} = \begin{pmatrix} a_2b_1 \\ -\frac{1}{2}(\sqrt{3}a_1b_2 + a_2b_2) \\ \frac{1}{2}(\sqrt{3}a_1b_3 - a_2b_3) \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_1b_1 \\ \frac{1}{2}(\sqrt{3}a_2b_2 - a_1b_2) \\ -\frac{1}{2}(\sqrt{3}a_2b_3 + a_1b_3) \end{pmatrix}_{\mathbf{3'}} , \tag{A6}$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3'}} = \begin{pmatrix} a_1b_1 \\ \frac{1}{2}(\sqrt{3}a_2b_2 - a_1b_2) \\ -\frac{1}{2}(\sqrt{3}a_2b_3 + a_1b_3) \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_2b_1 \\ -\frac{1}{2}(\sqrt{3}a_1b_2 + a_2b_2) \\ \frac{1}{2}(\sqrt{3}a_1b_3 - a_2b_3) \end{pmatrix}_{\mathbf{3'}} , \tag{A7}$$

$$\begin{aligned} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} &= (a_1b_1 + a_2b_2 + a_3b_3)_{\mathbf{1}} \oplus \begin{pmatrix} \frac{1}{\sqrt{2}}(a_2b_2 - a_3b_3) \\ \frac{1}{\sqrt{6}}(-2a_1b_1 + a_2b_2 + a_3b_3) \end{pmatrix}_{\mathbf{2}} \\ &\oplus \begin{pmatrix} a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{\mathbf{3'}} , \end{aligned} \tag{A8}$$

$$\begin{aligned} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3'}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3'}} &= (a_1b_1 + a_2b_2 + a_3b_3)_{\mathbf{1}} \oplus \begin{pmatrix} \frac{1}{\sqrt{2}}(a_2b_2 - a_3b_3) \\ \frac{1}{\sqrt{6}}(-2a_1b_1 + a_2b_2 + a_3b_3) \end{pmatrix}_{\mathbf{2}} \\ &\oplus \begin{pmatrix} a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{\mathbf{3'}} , \end{aligned} \tag{A9}$$

$$\begin{aligned} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3'}} &= (a_1b_1 + a_2b_2 + a_3b_3)_{\mathbf{1'}} \oplus \begin{pmatrix} \frac{1}{\sqrt{6}}(2a_1b_1 - a_2b_2 - a_3b_3) \\ \frac{1}{\sqrt{2}}(a_2b_2 - a_3b_3) \end{pmatrix}_{\mathbf{2}} \\ &\oplus \begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 \end{pmatrix}_{\mathbf{3'}} . \end{aligned} \tag{A10}$$

More details are shown in the review [27].

Appendix B: Next-to-leading order

Parameters appeared in the down-type quark mass matrix with next-to-leading order are $\bar{\epsilon}_{ij}$. These are explicitly written as

$$
\begin{aligned}
\bar{\epsilon}_{11} &= y_{\Delta_b} a_5 a_{14} v_d + \bar{y}_{\Delta_{c_2}} a_1 a_5 v_d, \\
\bar{\epsilon}_{12} &= -\frac{1}{2} y_{\Delta_b} a_5 a_{14} v_d - \left[\frac{\sqrt{3}}{4}(\sqrt{3}-1)\bar{y}_{\Delta_{c_1}} - \frac{1}{4}(\sqrt{3}+1)\bar{y}_{\Delta_{c_2}} \right] a_1 a_5 v_d, \\
\bar{\epsilon}_{13} &= \left[\left\{ \frac{\sqrt{3}}{4}(\sqrt{3}-1) y_{\Delta_{a_1}} + \frac{1}{4}(\sqrt{3}+1) y_{\Delta_{a_2}} \right\} a_1 a_{13} - \frac{1}{2} y_{\Delta_b} a_5 a_{14} \right] v_d \\
&\quad + \left[\left\{ -\frac{\sqrt{3}}{4}(\sqrt{3}+1)\bar{y}_{\Delta_{c_1}} - \frac{1}{4}(\sqrt{3}-1)\bar{y}_{\Delta_{c_2}} \right\} a_1 a_5 + \frac{\sqrt{3}}{2} \bar{y}_{\Delta_d} a_{13} a_{14} \right] v_d, \\
\bar{\epsilon}_{21} &= \bar{y}_{\Delta_{c_1}} a_1 a_5 v_d, \\
\bar{\epsilon}_{22} &= \frac{\sqrt{3}}{2} y_{\Delta_b} a_5 a_{14} v_d - \left[\frac{1}{4}(\sqrt{3}-1)\bar{y}_{\Delta_{c_1}} + \frac{\sqrt{3}}{4}(\sqrt{3}+1)\bar{y}_{\Delta_{c_2}} \right] a_1 a_5 v_d, \\
\bar{\epsilon}_{23} &= \left[\left\{ -\frac{1}{4}(\sqrt{3}-1) y_{\Delta_{a_1}} + \frac{\sqrt{3}}{4}(\sqrt{3}+1) y_{\Delta_{a_2}} \right\} a_1 a_{13} - \frac{\sqrt{3}}{2} y_{\Delta_b} a_5 a_{14} \right] v_d \\
&\quad + \left[\left\{ \frac{1}{4}(\sqrt{3}+1)\bar{y}_{\Delta_{c_1}} - \frac{\sqrt{3}}{4}(\sqrt{3}-1)\bar{y}_{\Delta_{c_2}} \right\} a_1 a_5 - \frac{1}{2} \bar{y}_{\Delta_d} a_{13} a_{14} \right] v_d, \\
\bar{\epsilon}_{31} &= -y_{\Delta_e} a_5 a_9 v_d + \bar{y}_{\Delta_f} a_9 a_{13} v_d, \\
\bar{\epsilon}_{33} &= y_{\Delta_e} a_5 a_9 v_d.
\end{aligned}
\tag{B1}
$$

Appendix C: Lepton sector

The mass matrix of charged lepton becomes

$$
M_l = \begin{pmatrix} 0 & -3y_1\lambda a_9 v_{45}/\sqrt{2} & 0 \\ 0 & -3y_1\lambda a_9 v_{45}/\sqrt{6} & 0 \\ 0 & 0 & y_2 a_{13} v_d \end{pmatrix}, \tag{C1}
$$

then, masses are given as

$$
m_e^2 = 0\ , \quad m_\mu^2 = 6|\bar{y}_1 \lambda a_9|^2 v_d^2\ , \quad m_\tau^2 = |y_2|^2 a_{13}^2 v_d^2\ . \tag{C2}
$$

In the same way, the right-handed Majorana mass matrix of neutrinos is given by

$$
M_N = \begin{pmatrix} y_1^N \lambda^2 \bar{\Lambda} + y_2^N a_4 \Lambda & 0 & 0 \\ 0 & y_1^N \lambda^2 \bar{\Lambda} - y_2^N a_4 \Lambda & 0 \\ 0 & 0 & M \end{pmatrix}, \tag{C3}
$$

and the Dirac mass matrix of neutrinos is

$$M_D = y_1^D \lambda v_u \begin{pmatrix} 2a_5/\sqrt{6} & -a_5/\sqrt{6} & -a_5/\sqrt{6} \\ 0 & a_5/\sqrt{2} & -a_5/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} + y_2^D v_u \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_5 & a_5 & a_5 \end{pmatrix} . \tag{C4}$$

By using the seesaw mechanism $M_\nu = M_D^T M_N^{-1} M_D$, the left-handed Majorana neutrino mass matrix is written as

$$M_\nu = \begin{pmatrix} a+\frac{2}{3}b & a-\frac{1}{3}b & a-\frac{1}{3}b \\ a-\frac{1}{3}b & a+\frac{1}{6}b+\frac{1}{2}c & a+\frac{1}{6}b-\frac{1}{2}c \\ a-\frac{1}{3}b & a+\frac{1}{6}b-\frac{1}{2}c & a+\frac{1}{6}b+\frac{1}{2}c \end{pmatrix} , \tag{C5}$$

where

$$a = \frac{(y_2^D a_5 v_u)^2}{M}, \qquad b = \frac{(y_1^D a_5 v_u \lambda)^2}{y_1^N \lambda^2 \bar{\Lambda} + y_2^N a_4 \Lambda}, \qquad c = \frac{(y_1^D a_5 v_u \lambda)^2}{y_1^N \lambda^2 \bar{\Lambda} - y_2^N a_4 \Lambda}. \tag{C6}$$

It gives the tri-bimaximal mixing matrix $U_{\text{tri-bi}}$ and mass eigenvalues as follows:

$$U_{\text{tri-bi}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} ,$$

$$m_{\nu_1} = b \ , \qquad m_{\nu_2} = 3a \ , \qquad m_{\nu_3} = c \ . \tag{C7}$$

The next-to-leading terms of the superpotential are important to predict the deviation from the tri-bimaximal mixing of leptons. The relevant superpotential in the charged lepton sector is given at the next-to-leading order as

$$\begin{aligned} \Delta w_l = & \, y_{\Delta_a} (T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_1, \chi_2) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{\bar{5}}/\Lambda^2 \\ & + y_{\Delta_b} (T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes \chi_{14} \otimes H_{\bar{5}}/\Lambda^2 \\ & + y_{\Delta_c} (T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_1, \chi_2) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_{45}/\Lambda^2 \\ & + y_{\Delta_d} (T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_{14} \otimes H_{45}/\Lambda^2 \\ & + y_{\Delta_e} T_3 \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes H_{\bar{5}} \otimes /\Lambda^2 \\ & + y_{\Delta_f} T_3 \otimes (F_1, F_2, F_3) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{45} \otimes /\Lambda^2 \ . \end{aligned} \tag{C8}$$

By using this superpotential, we obtain the charged lepton mass matrix as

$$M_l \simeq \begin{pmatrix} \epsilon_{11} & \frac{\sqrt{3}m_\mu}{2} + \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \frac{m_\mu}{2} + \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & 0 & m_\tau + \epsilon_{33} \end{pmatrix} , \tag{C9}$$

where m_μ and m_τ are given in Eq. (C2) and ϵ_{ij}'s are given as relevant linear combinations of $a_k a_l$'s. The explicit forms of ϵ_{ij}'s are given by replacing $\bar{y}_{\Delta_i}/3$ with $-\bar{y}_{\Delta_i}$ in $\bar{\epsilon}_{ij}$, which are presented in Appendix B. The charged lepton is diagonalized by the left-handed mixing matrix U_E and the right-handed one V_E as

$$V_E^\dagger M_\ell U_E = M_\ell^{\text{diag}}, \tag{C10}$$

where $M_\ell^{\rm diag}$ is a diagonal matrix. These mixing matrices can be written by

$$\begin{aligned} V_E &= \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & \frac{\tilde{a}^2}{\lambda^2} & \tilde{a} \\ -\frac{\tilde{a}^2}{\lambda^2} - \tilde{a}^2 & 1 & \tilde{a} \\ -\tilde{a} + \frac{\tilde{a}^3}{\lambda^2} & -\tilde{a} - \frac{\tilde{a}^3}{\lambda^2} & 1 \end{pmatrix}, \\ U_E &= \begin{pmatrix} 1 & \frac{\tilde{a}}{\lambda} & \tilde{a} \\ -\frac{\tilde{a}}{\lambda} - \tilde{a}^2 & 1 & \tilde{a} \\ -\tilde{a} + \frac{\tilde{a}^2}{\lambda} & -\tilde{a} - \frac{\tilde{a}^2}{\lambda} & 1 \end{pmatrix}. \end{aligned} \tag{C11}$$

Taking the next-to-leading order, the electron has non-zero mass, namely

$$m_e^2 \simeq \tfrac{3}{2}\left(\tfrac{1}{6}\epsilon_{11}^2 - \tfrac{1}{\sqrt{3}}\epsilon_{11}\epsilon_{21} + \tfrac{1}{2}\epsilon_{21}^2\right) \simeq \mathcal{O}(\tilde{a}^4 v_d^2). \tag{C12}$$

Appendix D: Formulae for quark sector

Here we will give formulae for quark sector which are used in our analysis. The SUSY contribution by gluino-squark box diagram to the dispersive part of the effective Hamiltonian for $M - \bar{M}$ mixing ($M = K, B_d, B_s$) is given by [166, 176]

$$\begin{aligned} M_{12}^{M,\rm SUSY} &= -\frac{\alpha_S^2}{216 m_{\tilde{q}}^2}\frac{2}{3}M_M f_M^2 \Bigg[\left\{(\delta_d^{LL})_{ij}^2 + (\delta_d^{RR})_{ij}^2\right\}\left\{24 x f_6(x) + 66\tilde{f}_6(x)\right\} \\ &+ (\delta_d^{LL})_{ij}(\delta_d^{RR})_{ij}\left(\left\{384\left(\frac{M_M}{m_j+m_i}\right)^2 + 72\right\} x f_6(x) + \left\{-24\left(\frac{M_M}{m_j+m_i}\right)^2 + 36\right\}\tilde{f}_6(x)\right) \\ &+ \left\{(\delta_d^{LR})_{ij}^2 + (\delta_d^{RL})_{ij}^2\right\}\left\{-132\left(\frac{M_M}{m_j+m_i}\right)^2\right\} x f_6(x) \\ &+ (\delta_d^{LR})_{ij}(\delta_d^{RL})_{ij}\left\{-144\left(\frac{M_M}{m_j+m_i}\right)^2 - 84\right\}\tilde{f}_6(x)\Bigg], \end{aligned} \tag{D1}$$

where $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$ and the loop functions are defined as

$$f_6(x) = \frac{6(1+3x)\log x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5}, \tag{D2}$$

$$\tilde{f}_6(x) = \frac{6x(1+x)\log x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5}. \tag{D3}$$

For $M = K, B_d, B_s$ meson system, the generation indices of down-type quarks (i,j) correspond to $(i,j) = (1,2), (1,3), (2,3)$, respectively.

For $b \to s\gamma$ decay, the Branching Ratio (BR) is given by

$$\mathrm{BR}(b \to s\gamma) = \alpha_s^2 \alpha \frac{m_b^3 \tau_B}{81\pi^2 m_{\tilde{q}}^4}\left[\left|m_b G_3(x)(\delta_d^{LL})_{23} + m_{\tilde{g}} G_1(x)(\delta_d^{LR})_{23}\right|^2 + (L \leftrightarrow R)\right], \tag{D4}$$

where τ_B is the lifetime of the B meson, and the loop functions are defined as

$$G_1(x) = \frac{1+4x-5x^2+4x\log x+2x^2\log x}{2(x-1)^4}, \tag{D5}$$

$$G_3(x) = \frac{-1+9x+9x^2-17x^3+18x^2\log x+6x^3\log x}{12(x-1)^5}. \tag{D6}$$

The chromo EDM of the strange quark is given by [163]

$$d_s^C = c\frac{\alpha_s}{4\pi}\frac{m_{\tilde g}}{m_{\tilde q}^2}\left(-\frac{1}{3}N_1(x)-3N_2(x)\right)\mathrm{Im}[(\delta_d^{LL})_{23}(\delta_d^{LR})_{33}(\delta_d^{RR})_{32}], \tag{D7}$$

where c is the QCD correction. We take $c = 0.9$. The functions $N_1(x)$ and $N_2(x)$ are given as follows:

$$N_1(x) = \frac{3+44x-36x^2-12x^3+x^4+12x(2+3x)\log x}{6(x-1)^6}, \tag{D8}$$

$$N_2(x) = -\frac{10+9x-18x^2-x^3+3(1+6x+3x^2)\log x}{3(x-1)^6}. \tag{D9}$$

Appendix E: $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$

In the framework of SUSY, LFV effects originate from misalignment between fermion and sfermion mass eigenstates. Once non-vanishing off-diagonal elements of the slepton mass matrices are generated in the super-CKM basis, LFV rare decays like $\ell_i \to \ell_j\gamma$ are naturally induced by one-loop diagrams with the exchange of gauginos and sleptons. The decay $\ell_i \to \ell_j\gamma$ is described by the dipole operator and the corresponding amplitude reads [171, 172, 176–178]

$$T = m_{\ell_i}\epsilon^\lambda \overline{u}_j(p-q)[iq^\nu\sigma_{\lambda\nu}(A_LP_L + A_RP_R)]u_i(p)\,, \tag{E1}$$

where p and q are momenta of the initial lepton ℓ_i and of the photon, respectively, and $A_{L,R}$ are the two possible amplitudes in this process. The branching ratio of $\ell_i \to \ell_j\gamma$ can be written as follows:

$$\frac{\mathrm{BR}(\ell_i \to \ell_j\gamma)}{\mathrm{BR}(\ell_i \to \ell_j\nu_i\bar{\nu}_j)} = \frac{48\pi^3\alpha}{G_F^2}(|A_L^{ij}|^2 + |A_R^{ij}|^2)\,.$$

In the mass insertion approximation, it is found that [166]

$$\begin{aligned} A_L^{ij} &\simeq \frac{\alpha_2}{4\pi}\frac{\left(\delta_\ell^{LL}\right)_{ij}}{m_{\tilde\ell}^2}\tan\beta\left[\frac{\mu M_2}{(M_2^2-\mu^2)}\left(f_{2n}(x_2,x_\mu)+f_{2c}(x_2,x_\mu)\right)\right.\\ &\left.\quad+\tan^2\theta_W\,\mu M_1\left(\frac{f_{3n}(x_1)}{m_{\tilde\ell}^2}+\frac{f_{2n}(x_1,x_\mu)}{(\mu^2-M_1^2)}\right)\right]+\frac{\alpha_1}{4\pi}\;\frac{\left(\delta_\ell^{RL}\right)_{ij}}{m_{\tilde\ell}^2}\;\left(\frac{M_1}{m_{\ell_i}}\right)\;2\;f_{2n}(x_1)\;,\\ A_R^{ij} &\simeq \frac{\alpha_1}{4\pi}\left[\frac{\left(\delta_e^{RR}\right)_{ij}}{m_{\tilde\ell}^2}\mu M_1\tan\beta\left(\frac{f_{3n}(x_1)}{m_{\tilde\ell}^2}-\frac{2f_{2n}(x_1,x_\mu)}{(\mu^2-M_1^2)}\right)+2\frac{\left(\delta_e^{LR}\right)_{ij}}{m_{\tilde\ell}^2}\;\left(\frac{M_1}{m_{\ell_i}}\right)\;f_{2n}(x_1)\right]\;, \end{aligned} \tag{E2}$$

where θ_W is the weak mixing angle, $x_{1,2} = M_{1,2}^2/m_{\tilde{\ell}}^2$, $x_\mu = \mu^2/m_{\tilde{\ell}}^2$ and $f_{i(c,n)}(x,y) = f_{i(c,n)}(x) - f_{i(c,n)}(y)$. The loop functions f_i's are given explicitly as follows:

$$\begin{aligned} f_{2n}(x) &= \frac{-5x^2 + 4x + 1 + 2x(x+2)\log x}{4(1-x)^4} \ , \\ f_{3n}(x) &= \frac{1 + 9x - 9x^2 - x^3 + 6x(x+1)\log x}{3(1-x)^5} \ , \\ f_{2c}(x) &= \frac{-x^2 - 4x + 5 + 2(2x+1)\log x}{2(1-x)^4} \ . \end{aligned} \tag{E3}$$

Appendix F: Electron electric dipole moment

The mass insertion parameters also contribute to the electron EDM through one-loop exchange of binos/sleptons. The corresponding EDM is given as [166, 179, 180]

$$\begin{aligned} \frac{d_e}{e} =& -\frac{\alpha_1}{4\pi}\frac{M_1}{m_{\tilde{\ell}}^2}\Big\{ \mathrm{Im}[(\delta_\ell^{LR})_{1k}(\delta_e^{RR})_{k1} + (\delta_\ell^{LL})_{1k}(\delta_\ell^{LR})_{k1}]\, f_{3n}(x_1) + \mathrm{Im}[(\delta_\ell^{LL})_{1k}(\delta_\ell^{LR})_{kl}(\delta_e^{RR})_{l1} \\ &+ (\delta_\ell^{LR})_{1k}(\delta_e^{RR})_{kl}(\delta_e^{RR})_{l1} + (\delta_\ell^{LL})_{1k}(\delta_\ell^{LL})_{kl}(\delta_\ell^{LR})_{l1}]\, f_{4n}(x_1)\Big\} \ , \end{aligned} \tag{F1}$$

where $k,l = 2,3$, $(\delta_\ell^{LR})_{33} = -m_\tau(A_\tau + \mu\tan\beta)/m_{\tilde{\ell}}^2$, and the loop function f_{4n} is given as

$$f_{4n}(x) = \frac{-3 - 44x + 36x^2 + 12x^3 - x^4 - 12x(3x+2)\log x}{6(1-x)^6} \ . \tag{F2}$$

Since components $(i,3)$ and $(3,i)$ of δ_e^{RR} are much larger compared to others in our model, dominant terms are given as

$$\frac{d_e}{e} \approx -\frac{\alpha_1}{4\pi}\frac{M_1}{m_{\tilde{\ell}}^2}\ \Big\{\ \mathcal{O}(\frac{m_e}{m_{\tilde{\ell}}}a_1)\, f_{3n}(x_1) + \mathcal{O}(\frac{m_\tau}{m_{\tilde{\ell}}}(1 + \frac{\mu\tan\beta}{m_{\tilde{\ell}}})a_1\tilde{a}^2)\, f_{4n}(x_1)\Big\}. \tag{F3}$$

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